

Contents

- Preface** vii
- 1 Introduction** 1
 - 1.1 The four problems $\mathbf{P}, \mathbf{P}_d, \mathbf{I}, \mathbf{I}_d$ 2
 - 1.2 Summary of content 4
- Part I Linear Integration and Linear Programming**
- 2 The Linear Integration Problem I** 9
 - 2.1 Introduction 9
 - 2.2 Primal methods 11
 - 2.3 A dual approach 15
 - 2.4 A residue algorithm for problem \mathbf{I}^* 18
 - 2.5 Notes 29
- 3 Comparing the Continuous Problems \mathbf{P} and \mathbf{I}** 31
 - 3.1 Introduction 31
 - 3.2 Comparing $\mathbf{P}, \mathbf{P}^*, \mathbf{I}$, and \mathbf{I}^* 33
 - 3.3 Notes 37
- Part II Linear Counting and Integer Programming**
- 4 The Linear Counting Problem \mathbf{I}_d** 41
 - 4.1 Introduction 41
 - 4.2 A primal approach: Barvinok's counting algorithm 42
 - 4.3 A dual approach 45
 - 4.4 Inversion of the \mathbb{Z} -transform by residues 48
 - 4.5 An algebraic method 52
 - 4.6 A simple explicit formula 65
 - 4.7 Notes 69

5	Relating the Discrete Problems P_d and I_d with P	71
5.1	Introduction	71
5.2	Comparing the dual problems I^* and I_d^*	72
5.3	A dual comparison of P and P_d	73
5.4	Proofs	77
5.5	Notes	79
Part III Duality		
6	Duality and Gomory Relaxations	83
6.1	Introduction	83
6.2	Gomory relaxations	84
6.3	Brion and Vergne's formula and Gomory relaxations	86
6.4	The Knapsack Problem	94
6.5	A dual of P_d	96
6.6	Proofs	99
6.7	Notes	106
7	Barvinok's Counting Algorithm and Gomory Relaxations	107
7.1	Introduction	107
7.2	Solving P_d via Barvinok's counting algorithm	108
7.3	The link with Gomory relaxations	111
7.4	Notes	112
8	A Discrete Farkas Lemma	115
8.1	Introduction	115
8.2	A discrete Farkas lemma	116
8.3	A discrete theorem of the alternative	125
8.4	The knapsack equation	127
8.5	Notes	129
9	The Integer Hull of a Convex Rational Polytope	131
9.1	Introduction	131
9.2	The integer hull	132
9.3	Notes	137
10	Duality and Superadditive Functions	139
10.1	Introduction	139
10.2	Preliminaries	140
10.3	Duality and superadditivity	142
10.4	Notes	147

Appendix Legendre–Fenchel, Laplace, Cramer, and \mathbb{Z} Transforms	149
A.1 The Legendre–Fenchel transform	149
A.2 Laplace transform	151
A.3 The \mathbb{Z} -transform	155
A.4 Notes	157
References	159
Glossary	165
Index	167