
CONTENTS

Every chapter has sections devoted to: notation, behavior, definitions, special cases, intrarelations, expansions, particular values, numerical values, limits and approximations, operations of the calculus, complex argument, generalizations, and cognate functions. In addition, each chapter has the special features itemized below its title.

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