
Contents

Preface	vii
Contents	xiii
List of Figures	xix
List of Tables	xxvii
1 Introduction	1
1.1 Outline	3
1.2 A note on programming	5
1.3 Symbols used throughout the book	6
2 Probability Theory and Classical Statistics	9
2.1 Rules of probability	9
2.2 Probability distributions in general	12
2.2.1 Important quantities in distributions	17
2.2.2 Multivariate distributions	19
2.2.3 Marginal and conditional distributions	23
2.3 Some important distributions in social science	25
2.3.1 The binomial distribution	25
2.3.2 The multinomial distribution	27
2.3.3 The Poisson distribution	28
2.3.4 The normal distribution	29
2.3.5 The multivariate normal distribution	30
2.3.6 t and multivariate t distributions	33
2.4 Classical statistics in social science	33
2.5 Maximum likelihood estimation	35
2.5.1 Constructing a likelihood function	36
2.5.2 Maximizing a likelihood function	38
2.5.3 Obtaining standard errors	39

2.5.4	A normal likelihood example	41
2.6	Conclusions	44
2.7	Exercises	44
2.7.1	Probability exercises	44
2.7.2	Classical inference exercises	45
3	Basics of Bayesian Statistics	47
3.1	Bayes' Theorem for point probabilities	47
3.2	Bayes' Theorem applied to probability distributions	50
3.2.1	Proportionality	51
3.3	Bayes' Theorem with distributions: A voting example	53
3.3.1	Specification of a prior: The beta distribution	54
3.3.2	An alternative model for the polling data: A gamma prior/ Poisson likelihood approach	60
3.4	A normal prior–normal likelihood example with σ^2 known	62
3.4.1	Extending the normal distribution example	65
3.5	Some useful prior distributions	68
3.5.1	The Dirichlet distribution	69
3.5.2	The inverse gamma distribution	69
3.5.3	Wishart and inverse Wishart distributions	70
3.6	Criticism against Bayesian statistics	70
3.7	Conclusions	73
3.8	Exercises	74
4	Modern Model Estimation Part 1: Gibbs Sampling	77
4.1	What Bayesians want and why	77
4.2	The logic of sampling from posterior densities	78
4.3	Two basic sampling methods	80
4.3.1	The inversion method of sampling	81
4.3.2	The rejection method of sampling	84
4.4	Introduction to MCMC sampling	88
4.4.1	Generic Gibbs sampling	88
4.4.2	Gibbs sampling example using the inversion method	89
4.4.3	Example repeated using rejection sampling	93
4.4.4	Gibbs sampling from a real bivariate density	96
4.4.5	Reversing the process: Sampling the parameters <i>given</i> the data	100
4.5	Conclusions	103
4.6	Exercises	105
5	Modern Model Estimation Part 2: Metroplis–Hastings Sampling	107
5.1	A generic MH algorithm	108
5.1.1	Relationship between Gibbs and MH sampling	113

5.2 Example: MH sampling when conditional densities are difficult to derive 115

5.3 Example: MH sampling for a conditional density with an unknown form 118

5.4 Extending the bivariate normal example: The full multiparameter model 121

 5.4.1 The conditionals for μ_x and μ_y 122

 5.4.2 The conditionals for σ_x^2 , σ_y^2 , and ρ 123

 5.4.3 The complete MH algorithm 124

 5.4.4 A matrix approach to the bivariate normal distribution problem 126

5.5 Conclusions 128

5.6 Exercises 129

6 Evaluating Markov Chain Monte Carlo Algorithms and Model Fit 131

6.1 Why evaluate MCMC algorithm performance? 132

6.2 Some common problems and solutions 132

6.3 Recognizing poor performance 135

 6.3.1 Trace plots 135

 6.3.2 Acceptance rates of MH algorithms 141

 6.3.3 Autocorrelation of parameters 146

 6.3.4 “ \hat{R} ” and other calculations 147

6.4 Evaluating model fit 153

 6.4.1 Residual analysis 154

 6.4.2 Posterior predictive distributions 155

6.5 Formal comparison and combining models 159

 6.5.1 Bayes factors 159

 6.5.2 Bayesian model averaging 161

6.6 Conclusions 163

6.7 Exercises 163

7 The Linear Regression Model 165

7.1 Development of the linear regression model 165

7.2 Sampling from the posterior distribution for the model parameters 168

 7.2.1 Sampling with an MH algorithm 168

 7.2.2 Sampling the model parameters using Gibbs sampling . . 169

7.3 Example: Are people in the South “nicer” than others? 174

 7.3.1 Results and comparison of the algorithms 175

 7.3.2 Model evaluation 178

7.4 Incorporating missing data 182

 7.4.1 Types of missingness 182

 7.4.2 A generic Bayesian approach when data are MAR: The “niceness” example revisited 186

7.5	Conclusions	191
7.6	Exercises	192
8	Generalized Linear Models	193
8.1	The dichotomous probit model	195
8.1.1	Model development and parameter interpretation	195
8.1.2	Sampling from the posterior distribution for the model parameters	198
8.1.3	Simulating from truncated normal distributions	200
8.1.4	Dichotomous probit model example: Black–white differences in mortality	206
8.2	The ordinal probit model	217
8.2.1	Model development and parameter interpretation	218
8.2.2	Sampling from the posterior distribution for the parameters	220
8.2.3	Ordinal probit model example: Black–white differences in health	223
8.3	Conclusions	228
8.4	Exercises	229
9	Introduction to Hierarchical Models	231
9.1	Hierarchical models in general	232
9.1.1	The voting example redux	233
9.2	Hierarchical linear regression models	240
9.2.1	Random effects: The random intercept model	241
9.2.2	Random effects: The random coefficient model	251
9.2.3	Growth models	256
9.3	A note on fixed versus random effects models and other terminology	264
9.4	Conclusions	268
9.5	Exercises	269
10	Introduction to Multivariate Regression Models	271
10.1	Multivariate linear regression	271
10.1.1	Model development	271
10.1.2	Implementing the algorithm	275
10.2	Multivariate probit models	277
10.2.1	Model development	278
10.2.2	Step 2: Simulating draws from truncated multivariate normal distributions	283
10.2.3	Step 3: Simulation of thresholds in the multivariate probit model	289
10.2.4	Step 5: Simulating the error covariance matrix	295
10.2.5	Implementing the algorithm	297
10.3	A multivariate probit model for generating distributions	303

10.3.1	Model specification and simulation	307
10.3.2	Life table generation and other posterior inferences	310
10.4	Conclusions	315
10.5	Exercises	317
11	Conclusion	319
A	Background Mathematics	323
A.1	Summary of calculus	323
A.1.1	Limits	323
A.1.2	Differential calculus	324
A.1.3	Integral calculus	326
A.1.4	Finding a general rule for a derivative	329
A.2	Summary of matrix algebra	330
A.2.1	Matrix notation	330
A.2.2	Matrix operations	331
A.3	Exercises	335
A.3.1	Calculus exercises	335
A.3.2	Matrix algebra exercises	335
B	The Central Limit Theorem, Confidence Intervals, and Hypothesis Tests	337
B.1	A simulation study	337
B.2	Classical inference	338
B.2.1	Hypothesis testing	339
B.2.2	Confidence intervals	342
B.2.3	Some final notes	344
	References	345
	Index	353