

Contents

1	Newton's Law of Universal Gravitation	1
1.1	The Force Law of Gravitation	1
1.2	Newton's Law of Gravitation in Its Local Form	2
1.3	Tidal Forces	5
1.4	The Principle of Equivalence	9
1.5	The General Principle of Relativity	10
1.6	The Covariance Principle	11
1.7	Mach's Principle	12
	Problems	13
	References	16
2	The Special Theory of Relativity	17
2.1	Coordinate Systems and Minkowski Diagrams	17
2.2	Synchronization of Clocks	19
2.3	The Doppler Effect	20
2.4	Relativistic Time Dilation	22
2.5	The Relativity of Simultaneity	23
2.6	The Lorentz Contraction	26
2.7	The Lorentz Transformation	27
2.8	Lorentz-Invariant Interval	29
2.9	The Twin Paradox	32
2.10	Hyperbolic Motion	33
2.11	Energy and Mass	36
2.12	Relativistic Increase of Mass	37
2.13	Tachyons	38
2.14	Magnetism as a Relativistic Second-Order Effect	39
	Problems	42
	Reference	49
3	Vectors, Tensors and Forms	51
3.1	Vectors	51
3.1.1	4-Vectors	52
3.1.2	Tangent Vector Fields and Coordinate Vectors	54

3.1.3	Coordinate Transformations	58
3.1.4	Structure Coefficients	61
3.2	Tensors	63
3.2.1	Transformation of Tensor Components	65
3.2.2	Transformation of Basis 1-Forms	65
3.2.3	The Metric Tensor	66
3.3	Forms	69
	Problems	71
	Reference	75
4	Accelerated Reference Frames	77
4.1	Rotating Reference Frames	77
4.1.1	The Spatial Metric Tensor	77
4.1.2	Angular Acceleration in the Rotating Frame	81
4.1.3	Gravitational Time Dilation	83
4.1.4	Path of Photons Emitted from Axes in the Rotating Reference Frame (RF)	84
4.1.5	The Sagnac Effect	85
4.2	Hyperbolically Accelerated Reference Frames	86
	Problems	92
5	Covariant Differentiation	97
5.1	Differentiation of Forms	97
5.1.1	Exterior Differentiation	97
5.1.2	Covariant Derivative	99
5.2	The Christoffel Symbols	101
5.3	Geodesic Curves	104
5.4	The Covariant Euler–Lagrange Equations	105
5.5	Application of the Lagrangian Formalism to Free Particles	107
5.5.1	Equation of Motion from Lagrange’s Equations	107
5.5.2	Geodesic World Lines in Spacetime	108
5.5.3	Gravitational Doppler Effect	117
5.6	The Koszul Connection	119
5.7	Connection Coefficients $\Gamma_{\mu\nu}^{\alpha}$ and Structure Coefficients $c_{\mu\nu}^{\alpha}$	121
5.8	Covariant Differentiation of Vectors, Forms and Tensors	122
5.8.1	Covariant Differentiation of a Vector in an Arbitrary Basis	122
5.8.2	Covariant Differentiation of Forms	123
5.8.3	Generalization for Tensors of Higher Rank	124
5.9	The Cartan Connection	125
	Problems	128
	Reference	132

- 6 Curvature** 133
 - 6.1 The Riemann Curvature Tensor 133
 - 6.2 Differential Geometry of Surfaces 137
 - 6.2.1 Surface Curvature Using the Cartan Formalism 140
 - 6.3 The Ricci Identity 141
 - 6.4 Bianchi’s 1st Identity 142
 - 6.5 Bianchi’s 2nd Identity 143
 - Problems 144

- 7 Einstein’s Field Equations** 149
 - 7.1 Energy–Momentum Conservation 149
 - 7.1.1 Newtonian Fluid 149
 - 7.1.2 Perfect Fluids 151
 - 7.2 Einstein’s Curvature Tensor 151
 - 7.3 Einstein’s Field Equations 152
 - 7.4 The “Geodesic Postulate” as a Consequence of the Field Equations . 154
 - Problems 155

- 8 The Schwarzschild Spacetime** 159
 - 8.1 Schwarzschild’s Exterior Solution 159
 - 8.2 Radial Free Fall in Schwarzschild Spacetime 163
 - 8.3 Light Cones in Schwarzschild Spacetime 165
 - 8.4 Analytical Extension of the Schwarzschild Coordinates 167
 - 8.5 Embedding of the Schwarzschild Metric 169
 - 8.6 Deceleration of Light 170
 - 8.7 Particle Trajectories in Schwarzschild 3-Space 171
 - 8.7.1 Motion in the Equatorial Plane 173
 - 8.8 Classical Tests of Einstein’s General Theory of Relativity 175
 - 8.8.1 The Hafele–Keating Experiment 175
 - 8.8.2 Mercury’s Perihelion Precession 176
 - 8.8.3 Deflection of Light 177
 - Problems 179
 - Reference 185

- 9 Black Holes** 187
 - 9.1 “Surface Gravity”: Gravitational Acceleration on the Horizon of a Black Hole 187
 - 9.2 Hawking Radiation: Radiation from a Black Hole 188
 - 9.3 Rotating Black Holes: The Kerr Metric 189
 - 9.3.1 Zero-Angular-Momentum-Observers 190
 - 9.3.2 Does the Kerr Space Have a Horizon? 191
 - Problems 192

10	Schwarzschild's Interior Solution	199
10.1	Newtonian Incompressible Star	199
10.2	The Pressure Contribution to the Gravitational Mass of a Static, Spherically Symmetric System	201
10.3	The Tolman–Oppenheimer–Volkoff Equation	202
10.4	An Exact Solution for Incompressible Stars – Schwarzschild's Interior Solution	204
	Problems	205
11	Cosmology	207
11.1	Comoving Coordinate System	207
11.2	Curvature Isotropy – The Robertson–Walker Metric	208
11.3	Cosmic Dynamics	209
11.3.1	Hubble's Law	209
11.3.2	Cosmological Redshift of Light	209
11.3.3	Cosmic Fluids	211
11.3.4	Isotropic and Homogeneous Universe Models	212
11.4	Some Cosmological Models	215
11.4.1	Radiation-Dominated Model	215
11.4.2	Dust-Dominated Model	216
11.4.3	Transition from Radiation – To Matter-Dominated Universe	220
11.4.4	Friedmann–Lemaître Model	221
11.5	Inflationary Cosmology	230
11.5.1	Problems with the Big Bang Models	230
11.5.2	Cosmic Inflation	232
	Problems	237
	Reference	244
	Index	245