

# Contents

<b>Preface</b>	<b>vii</b>
<b>I Foundations</b>	<b>1</b>
<b>1 The Karmarkar Revolution</b>	<b>3</b>
1.1 Classical Portrait of the Field . . . . .	3
1.2 Modern Portrait of the Field . . . . .	6
1.3 Overview of Monograph . . . . .	8
1.4 Notes . . . . .	10
<b>2 The Newton–Cauchy Method</b>	<b>11</b>
2.1 Introduction . . . . .	11
2.1.1 Informational Complexity Assumptions . . . . .	12
2.2 Model-Based Perspective . . . . .	13
2.2.1 Newton . . . . .	13
2.2.2 Quasi-Newton . . . . .	15
2.2.3 Limited Memory . . . . .	15
2.2.4 Modified Cauchy . . . . .	16
2.2.5 Summary . . . . .	16
2.3 Metric-Based Perspective . . . . .	16
2.3.1 Cauchy . . . . .	17
2.3.2 Variable Metric . . . . .	17

2.3.3	Limited Memory . . . . .	18
2.3.4	Modified Newton . . . . .	21
2.3.5	Summary . . . . .	21
2.4	Newton–Cauchy Framework . . . . .	22
2.4.1	Positive Definite Quadratic Models . . . . .	22
2.4.2	The NC Method . . . . .	23
2.5	Notes . . . . .	24
<b>3</b>	<b>Euler–Newton and Lagrange–NC Methods</b>	<b>25</b>
3.1	Introduction . . . . .	25
3.2	The EN Method . . . . .	28
3.2.1	Definitions and Path Existence . . . . .	29
3.2.2	Embedding Algorithms . . . . .	30
3.2.3	EN Algorithms . . . . .	31
3.2.4	Algorithmic Enhancements . . . . .	32
3.3	The LNC Method . . . . .	35
3.3.1	An Alternative View . . . . .	36
3.3.2	Lagrangian Globalization . . . . .	38
3.3.3	An Illustrative Algorithm . . . . .	40
3.3.4	LNC Algorithms . . . . .	42
3.4	Notes . . . . .	45
<b>II</b>	<b>Lessons from One Dimension</b>	<b>47</b>
<b>4</b>	<b>A Misleading Paradigm</b>	<b>49</b>
4.1	The Unidimensional Potential Function . . . . .	50
4.2	Globally Convergent Algorithms . . . . .	51
4.3	Rapidly Convergent Local Algorithms . . . . .	52
4.4	Practical Hybrid Algorithms . . . . .	54
4.5	Summary . . . . .	54
<b>5</b>	<b>CG and the Line Search</b>	<b>57</b>
5.1	The Linear CG Algorithm . . . . .	58
5.2	Nonlinear CG-Related Algorithms . . . . .	60
5.2.1	The Nonlinear CG Algorithm . . . . .	60
5.2.2	The PARTAN Algorithm . . . . .	60
5.2.3	The Heavy-Ball Algorithm . . . . .	61
5.3	The Key Role of the Line Search . . . . .	62
5.4	A Line-Search Fortran Routine . . . . .	63
5.5	CG: Whither or Wither? . . . . .	66
5.5.1	Other Nonlinear CG Algorithms . . . . .	66
5.5.2	A New Two-Parameter CG Family . . . . .	68
5.5.3	Illustration of Performance . . . . .	68
5.5.4	Conclusions . . . . .	70

5.6	Notes . . . . .	72
<b>6</b>	<b>Gilding the Nelder–Mead Lily</b>	<b>73</b>
6.1	Golden-Section Search . . . . .	74
6.2	The Nelder–Mead Algorithm . . . . .	75
6.3	Gilding the Lily . . . . .	76
6.4	Numerical Experiments . . . . .	78
6.5	Summary . . . . .	79
6.6	Notes . . . . .	79
<b>III</b>	<b>Choosing the Right Diagonal Scale</b>	<b>81</b>
<b>7</b>	<b>Historical Parallels</b>	<b>83</b>
7.1	The Simplex Algorithm . . . . .	84
7.2	The Affine-Scaling Algorithm . . . . .	86
7.3	Numerical Illustration . . . . .	87
7.4	Historical Parallels . . . . .	91
7.5	Notes . . . . .	93
<b>8</b>	<b>LP from the Newton–Cauchy Perspective</b>	<b>95</b>
8.1	Primal Affine Scaling . . . . .	96
8.1.1	Model-Based Perspective . . . . .	96
8.1.2	Metric-Based Perspective . . . . .	97
8.1.3	NC Perspective . . . . .	97
8.1.4	Equivalence . . . . .	98
8.2	Dual Affine Scaling . . . . .	98
8.3	Primal–Dual Affine Scaling . . . . .	99
8.4	The Central Path . . . . .	101
8.5	Convergence and Implementation . . . . .	101
8.6	Notes . . . . .	102
<b>9</b>	<b>Diagonal Metrics and the QC Method</b>	<b>103</b>
9.1	A Quasi-Cauchy Algorithm . . . . .	105
9.1.1	Descent Direction . . . . .	105
9.1.2	Line Search . . . . .	106
9.1.3	Diagonal Metric . . . . .	106
9.1.4	Numerical Illustration . . . . .	107
9.2	The QC Method . . . . .	108
9.2.1	Quasi-Gradient . . . . .	109
9.2.2	Line Search . . . . .	109
9.2.3	Diagonal Updating . . . . .	109
9.3	Extension of the NC Framework . . . . .	111
9.4	Notes . . . . .	113

<b>IV</b>	<b>Linear Programming Post-Karmarkar</b>	<b>115</b>
<b>10</b>	<b>LP from the Euler–Newton Perspective</b>	<b>117</b>
10.1	The Parameterized Homotopy System . . . . .	118
10.1.1	Interior Paths . . . . .	118
10.1.2	Noninterior Paths . . . . .	122
10.2	Path-Following Building Blocks . . . . .	124
10.2.1	Notation . . . . .	125
10.2.2	Euler Predictor . . . . .	125
10.2.3	Second-Order Corrector . . . . .	127
10.2.4	Newton Corrector . . . . .	127
10.2.5	Restarts . . . . .	128
10.3	Numerical Illustration . . . . .	129
10.4	Path-Following Algorithms . . . . .	132
10.4.1	Notation . . . . .	132
10.4.2	Strategies . . . . .	133
10.4.3	Measures and Targets . . . . .	135
10.4.4	Complexity Results . . . . .	138
10.5	Mehrotra’s Implementation . . . . .	140
10.6	Summary . . . . .	144
10.7	Notes . . . . .	145
<b>11</b>	<b>Log-Barrier Transformations</b>	<b>147</b>
11.1	Derivation . . . . .	148
11.1.1	Primal . . . . .	148
11.1.2	Dual . . . . .	149
11.1.3	Primal–Dual . . . . .	150
11.2	Special Cases . . . . .	151
11.2.1	Primal . . . . .	151
11.2.2	Dual . . . . .	152
11.2.3	Primal–Dual . . . . .	152
11.3	Discussion . . . . .	152
11.4	Notes . . . . .	153
<b>12</b>	<b>Karmarkar Potentials and Algorithms</b>	<b>155</b>
12.1	Derivation . . . . .	156
12.1.1	Notation . . . . .	156
12.1.2	Unweighted Case . . . . .	156
12.1.3	Weighted Case . . . . .	158
12.2	A Potential-Reduction Algorithm . . . . .	158
12.3	Discussion . . . . .	161
12.4	Concluding Remarks . . . . .	163
12.5	Notes . . . . .	163

<b>V</b>	<b>Algorithmic Science</b>	<b>165</b>
<b>13</b>	<b>Algorithmic Principles</b>	<b>167</b>
13.1	Introduction . . . . .	167
13.2	Duality . . . . .	168
13.2.1	Complementing Operation . . . . .	168
13.2.2	The Broyden Family . . . . .	170
13.2.3	Self-Complementary Updates . . . . .	171
13.3	Invariance . . . . .	171
13.4	Symmetry . . . . .	172
13.5	Conservation . . . . .	174
13.5.1	Variational Principles . . . . .	174
13.5.2	Continuity . . . . .	175
13.6	The “Essentialist” View . . . . .	175
13.7	Notes . . . . .	177
<b>14</b>	<b>Multialgorithms: A New Paradigm</b>	<b>179</b>
14.1	Background . . . . .	179
14.1.1	Traditional UM Algorithms . . . . .	179
14.1.2	Evolutionary GUM Algorithms . . . . .	181
14.1.3	Optimizing an Optimizer . . . . .	183
14.2	Population-Thinking and Multialgorithms . . . . .	186
14.3	CG Multialgorithms: A Case Study . . . . .	187
14.3.1	Fixed-Population Multialgorithms . . . . .	188
14.3.2	Evolving-Population Multialgorithms . . . . .	190
14.3.3	Computational Experiments . . . . .	190
14.4	The Multialgorithms Paradigm . . . . .	195
14.4.1	Terminology . . . . .	195
14.4.2	Discussion . . . . .	196
14.5	Parallel Computing Platform . . . . .	198
14.6	Other Topics . . . . .	199
14.6.1	Convergence Analysis . . . . .	199
14.6.2	A Performance Evaluation Environment . . . . .	200
14.6.3	Further Areas of Application . . . . .	200
14.7	Recapitulation . . . . .	202
14.8	Notes . . . . .	203
<b>15</b>	<b>An Emerging Discipline</b>	<b>205</b>
15.1	Background . . . . .	206
15.2	What Is an Algorithm? . . . . .	208
15.3	Models of Computation . . . . .	211
15.3.1	Rational-Number Model/RaM . . . . .	211
15.3.2	Real-Number Model/GaM . . . . .	212
15.3.3	Floating-Point Model/FRaM . . . . .	214
15.4	Conceptual Algorithms . . . . .	215

15.5 Implementable Algorithms . . . . .	216
15.5.1 Level 1: Mathematical . . . . .	217
15.5.2 Level 2: Numerical . . . . .	219
15.5.3 Level 3: Quality Software . . . . .	220
15.5.4 Interplay Between Levels . . . . .	221
15.6 Algorithmic Science . . . . .	222
15.7 Notes . . . . .	223
<b>References</b>	<b>225</b>
<b>Index</b>	<b>251</b>