

Contents

1. Transformations of the Plane	1
1.1 Introduction	1
1.2 Translations	5
1.3 Scaling about the Origin	6
1.4 Reflections	8
1.5 Rotation about the Origin	9
1.6 Shears	11
1.7 Concatenation of Transformations	13
1.8 Applications	15
1.8.1 Instancing	15
1.8.2 Robotics	17
2. Homogeneous Coordinates and Transformations of the Plane	19
2.1 Introduction	19
2.1.1 Homogeneous Coordinates	21
2.2 Points at Infinity	23
2.3 Visualization of the Projective Plane	24
2.3.1 Line Model of the Projective Plane	24
2.3.2 Spherical Model of the Projective Plane	26
2.4 Transformations in Homogeneous Coordinates	27
2.4.1 Translations	27
2.4.2 Scaling about the Origin	28
2.4.3 Rotation about the Origin	29
2.5 Concatenation of Transformations	30
2.5.1 Inverse Transformations	31

2.5.2	Rotation about an Arbitrary Point	33
2.5.3	Reflection in an Arbitrary Line	34
2.6	Applications	36
2.6.1	Instancing	36
2.6.2	Device Coordinate Transformation	37
2.7	Point and Line Geometry in Homogeneous Coordinates	38
3.	Homogeneous Coordinates and Transformations of Space ...	41
3.1	Homogeneous Coordinates	41
3.2	Transformations of Space	42
3.2.1	Translations	42
3.2.2	Scalings and Reflections	43
3.2.3	Rotations about the Coordinate Axes	43
3.2.4	Rotation about an Arbitrary Line	45
3.2.5	Reflection in an Arbitrary Plane	47
3.3	Applications	49
3.3.1	Computer-aided Design	49
3.3.2	Orientation of a Rigid Body	50
3.4	Geometric Methods for Lines and Planes in Space	52
3.5	Quaternions	56
4.	Projections and the Viewing Pipeline	67
4.1	Introduction	67
4.2	Projections of the Plane	68
4.3	Projections of Three-dimensional Space	72
4.4	The Viewplane Coordinate Mapping	76
4.5	The Viewing Pipeline	80
4.6	Classification of Projections	85
4.6.1	Classification of Parallel Projections	85
4.6.2	Classification of Perspective Projections	90
5.	Curves	95
5.1	Introduction	95
5.2	Curve Rendering	98
5.3	Parametric Curves	99
5.4	Arclength and Reparametrization	102
5.5	Application: Numerical Controlled Machining and Offsets	107
5.6	Conics	109
5.6.1	Classification of Conics	112
5.6.2	Conics in Standard Form	116
5.6.3	Intersections of a Conic with a Line	121
5.6.4	Parametrization of an Irreducible Conic	124

5.6.5	Converting from Parametric Form to Implicit Form	127
5.7	Conics in Space	130
5.8	Applications of Conics	132
6.	Bézier Curves I	135
6.1	Introduction	135
6.2	Bézier Curves of Low Degree	136
6.2.1	Linear Bézier Curves	136
6.2.2	Quadratic Bézier Curves	136
6.2.3	Cubic Bézier Curves	137
6.3	The Effect of Adjusting a Control Point	140
6.4	The General Bézier Curve	141
6.5	Properties of the Bernstein Polynomials	144
6.6	Convex Hulls	146
6.7	Properties of Bézier Curves	147
6.8	The de Casteljau Algorithm	151
6.9	Subdivision of a Bézier Curve	154
6.10	Applications	157
6.10.1	Rendering	157
6.10.2	Intersection of a Planar Bézier Curve and a Line	158
6.10.3	Intersection of Two Bézier Curves	159
7.	Bézier Curves II	161
7.1	Spatial Bézier Curves	161
7.2	Derivatives of Bézier Curves	162
7.3	Conversions Between Representations	166
7.4	Piecewise Bézier Curves	168
7.5	Rational Bézier Curves	175
7.5.1	Properties of Rational Bézier Curves	177
7.5.2	de Casteljau Algorithm for Rational Curves	180
7.5.3	Projections of Rational Bézier Curves	181
7.5.4	Derivatives of Rational Bézier Curves	185
8.	B-splines	187
8.1	Integral B-spline Curves	187
8.1.1	Properties of the B-spline Curve	194
8.1.2	B-spline Types	196
8.1.3	Application: Font Design	203
8.1.4	Application: Morphing or Soft Object Animation	203
8.1.5	The de Boor Algorithm	205
8.1.6	Derivatives of a B-spline	207
8.2	Non-uniform Rational B-Splines (NURBS)	212

8.2.1	Projections of NURBS Curves	214
8.2.2	Derivatives of NURBS	216
8.2.3	Rational de Boor Algorithm	218
8.3	Knot Insertion	221
9.	Surfaces	225
9.1	Introduction	225
9.2	Quadric Surfaces	228
9.2.1	Offset Surfaces	232
9.3	Bézier and B-spline Surfaces	234
9.3.1	Properties of Bézier and B-spline Surfaces	235
9.3.2	Derivatives of Bézier and B-spline Surfaces	238
9.4	Surface Constructions	241
9.4.1	Extruded Surfaces	241
9.4.2	Ruled Surfaces	242
9.4.3	Translationally Swept Surfaces	244
9.4.4	Surfaces of Revolution	245
9.5	Surface Subdivision	248
9.6	Skin and Loft Surfaces	251
9.7	Geometric Modelling and CAD	260
9.7.1	Wireframe Modeller	261
9.7.2	Surface Modeller	261
9.7.3	Constructive Solid Geometry (CSG) Modellers	261
9.7.4	Boundary Representations (B-rep)	263
10.	Curve and Surface Curvatures	267
10.1	Curvature of a Plane Curve	267
10.2	Curvature and Torsion of a Space Curve	275
10.3	Curvature of Bézier Curves	283
10.4	Surface Curvatures	285
11.	Rendering	297
11.1	Introduction	297
11.2	Colour	298
11.3	An Illumination Model for Reflected Light	299
11.3.1	Diffuse Reflection	300
11.3.2	Specular Reflection	302
11.3.3	Ambient Reflection	304
11.3.4	Attenuation	305
11.3.5	Total Intensity	305
11.4	Shading Algorithms	306
11.4.1	Flat Shading	307

11.4.2 Gouraud Shading	307
11.4.3 Phong Shading	309
11.5 Silhouettes	309
11.6 Shadows	320
Solutions	323
References	345
Index	347