

# Contents

<b>Part I. Preliminaries</b> . . . . .	1
<b>1. Introduction</b> . . . . .	4
1.1 PLL structure . . . . .	4
1.2 PLLs classification . . . . .	5
1.3 The weak points of the classical PLL theory. . . . .	6
1.4 Scope and methodology . . . . .	7
References . . . . .	10
<b>2. PLL components</b> . . . . .	13
2.1 PLL structure . . . . .	13
2.2 Voltage controlled oscillator. . . . .	13
2.2-1 VCO representation . . . . .	14
2.2-2 Transformation of the VCO equation . . . . .	16
2.2-3 Averaged VCO equations . . . . .	19
2.3 Phase detector . . . . .	21
2.4 Low-pass filter. . . . .	23
2.4-1 Phase lead-lag filter . . . . .	23
2.4-2 PI controller . . . . .	25
2.5 Conclusions . . . . .	27
References . . . . .	28
<b>Part II. First order PLL.</b> . . . . .	31
<b>3. Introduction to first order PLL</b> . . . . .	33
3.1 Representation of the first order loop. General form. . . . .	33
3.2 Physical interpretation of the PLL mechanism. . . . .	36
3.2-1 Main synchronization . . . . .	36
3.2-2 Third harmonic synchronization . . . . .	38
3.3 Representation of the first order PLL. Specific form . . . . .	38
3.4 Preliminary investigation . . . . .	40

3.5 Conclusions . . . . .	42
References . . . . .	43
<b>4. Main synchronization . . . . .</b>	<b>45</b>
4.1 Local stability . . . . .	45
4.1-1 Equilibrium points . . . . .	45
4.1-2 Effect of parameters on equilibrium points . . . . .	47
4.1-3 Comment . . . . .	51
4.1-4 Local stability conditions . . . . .	51
4.2 Hold-in range . . . . .	53
4.3 Tracking range . . . . .	57
4.4 Global dynamic behaviour . . . . .	63
4.4-1 Transformation of the averaged equations . . . . .	63
4.4-2 Capture phenomenon . . . . .	64
4.4-3 Detuning . . . . .	72
4.5 Conclusions . . . . .	74
References . . . . .	74
<b>5. Third harmonic synchronization . . . . .</b>	<b>75</b>
5.1 Local stability . . . . .	75
5.1-1 Equilibrium points . . . . .	76
5.1-2 Effect of parameters on equilibrium points . . . . .	77
5.1-3 Local stability conditions . . . . .	78
5.2 Hold-in range . . . . .	80
5.3 Tracking range . . . . .	81
5.4 Global dynamic behaviour . . . . .	83
5.4-1 Transformation of averaged equations. . . . .	83
5.4-2 Capture phenomenon . . . . .	83
5.4-3 Detuning . . . . .	84
5.5 Conclusions . . . . .	85
<b>Part III. Second order type-I PLL . . . . .</b>	<b>87</b>
<b>6. Introduction to second order type-I PLL . . . . .</b>	<b>89</b>
6.1 Representation of the second order type-II PLL . . . . .	90
6.1-1 Exact equations . . . . .	91
6.1-2 Averaged equations . . . . .	92
6.2 Preliminary investigation . . . . .	94
References . . . . .	99

<b>7. Third harmonic synchronization</b> . . . . .	101
7.1 Local stability . . . . .	101
7.1-1 Equilibrium points . . . . .	102
7.1-2 Local stability conditions . . . . .	102
7.1-3 Hold-in range . . . . .	104
7.2 Global stability . . . . .	105
7.3 Conclusions . . . . .	111
<b>8. Main synchronization</b> . . . . .	113
8.1 Local stability . . . . .	113
8.1-1 Equilibrium points . . . . .	113
8.1-2 Local stability conditions . . . . .	116
8.2 Hold-in range . . . . .	118
8.2-1 Effect of parameters on hold-in range. . . . .	118
8.3 Global stability . . . . .	123
8.3-1 Slow capture . . . . .	123
8.3-2 Mean capture range . . . . .	127
8.3-3 Effect of parameters on mean capture range . . . . .	133
8.3-4 Effect of initial conditions . . . . .	136
8.3-5 Comment . . . . .	136
8.4 Dynamic behaviour . . . . .	137
8.4-1 Qualitative analysis . . . . .	138
8.4-2 Effect of parameters on loop dynamics . . . . .	142
8.4-3 Quantitative analysis. Fast capture . . . . .	145
8.5 Tracking range . . . . .	148
8.5-1 Local tracking limits . . . . .	149
8.5-2 Global tracking limits . . . . .	150
8.6 Hold-in and capture ranges . . . . .	152
8.7 Effect of noise and disturbances . . . . .	154
8.8 Design criteria . . . . .	156
8.9 Conclusions . . . . .	159
References . . . . .	159
<b>Part IV. Second order type-II PLL</b> . . . . .	161
<b>9. Introduction to second order type-II PLL</b> . . . . .	163
9.1 Representation of the second order type-II PLL . . . . .	164
9.1-1 Exact equations . . . . .	164
9.1-2 Averaged equations . . . . .	166
9.2 Preliminary investigation . . . . .	168
9.3 Conclusions . . . . .	172
References . . . . .	172

**10. Main synchronization** . . . . . 173

- 10.1 Local stability of the unsaturated loop . . . . . 173
  - 10.1-1 Equilibrium points . . . . . 174
  - 10.1-2 Local stability conditions. . . . . 175
  - 10.1-3 Hold-in range. . . . . 177
  - 10.1-4 Effect of parameters on hold-in range . . . . . 178
- 10.2 Dynamic behaviour of the unsaturated loop . . . . . 186
  - 10.2-1 Qualitative analysis . . . . . 186
  - 10.2-2 Quantitative analysis . . . . . 189
- 10.3 Tracking range. . . . . 192
- 10.4 Conclusions . . . . . 198
- References . . . . . 198

**11. Third harmonic synchronization** . . . . . 199

- 11.1 Global stability . . . . . 199
- 11.2 Conclusions . . . . . 202

**Part V. Third order type-II PLL** . . . . . 203

**12. Introduction to third order type-II PLL** . . . . . 205

- 12.1 About the filters used in the third order loops . . . . . 205
- 12.2 Representation of the third order type-II PLL . . . . . 207
  - 12.1-1 Exact equations . . . . . 207
  - 12.1-2 Averaged equations . . . . . 209
- References . . . . . 211

**13. Main synchronization** . . . . . 213

- 13.1 Local stability of the unsaturated loop . . . . . 213
  - 13.1-1 Equilibrium points . . . . . 214
  - 13.1-2 Local stability conditions. . . . . 216
- 13.2 Hold-in range. . . . . 218
  - 13.2-1 Effect of loop parameters on hold-in range. . . 219
- 13.3 Tracking range. . . . . 224
- 13.4 Dynamic behaviour . . . . . 226
- 13.5 Conclusions . . . . . 229

**14. Third harmonic synchronization** . . . . . 231

- 14.1 Global stability . . . . . 231

**Part VI. Appendices . . . . . 237**

**I.** First order PLL. Averaged equations . . . . . 239

**II.** First order PLL. Equilibrium points of the main  
synchronization range . . . . . 243

**III.** Second order type-I PLL. Averaged equations. . . . . 247

**IV.** Second order type-I PLL. State equation of the low-pass  
filter for the capture range calculation. . . . . 253

**V.** Second order type-I PLL. Effect of initial condition on  
capture range . . . . . 255

**VI.** Second order type-II PLL. Averaged equations . . . . . 257

**VII.** Third order type-II PLL. Averaged equations . . . . . 263

**VIII.** Resonant voltage controlled oscillator. . . . . 269

**IX.** Vector phase detector . . . . . 273

**X.** Experimental verification . . . . . 275

**Index. . . . . 283**