

# CONTENTS

<b>Preface</b>	<b>v</b>
<b>Nomenclature</b>	<b>xi</b>
<b>0 Introduction</b>	<b>1</b>
0.1 The Conception of Rarefied Gas Dynamics .....	1
0.2 The Molecular Model of Gases .....	3
0.3 Mean Free Path of Molecules .....	4
0.4 Division of Flow Regimes .....	5
0.5 Nonequilibrium Phenomena and Rarefied Gas Dynamics .....	9
0.6 Similarity Criteria .....	13
References .....	18
<b>1 Molecular Structure and Energy States</b>	<b>21</b>
1.1 Diatomic Molecules .....	21
1.2 Energy Distribution of Molecules .....	30
1.2.1 Boltzmann's Relation .....	32
1.2.2 Calculation of The Number $\Omega$ of Microscopic States .....	34
1.2.3 Boltzmann Distribution .....	37
1.3 Internal Energy Distribution Functions .....	43
References .....	50
<b>2 Some Basic Concepts of Kinetic Theory</b>	<b>51</b>
2.1 The Velocity Distribution Function .....	51
2.2 Macroscopic Properties .....	53
2.3 Binary Elastic Collisions of Molecules .....	61
2.4 Collision Cross-sections and Molecule Models .....	69
2.4.1 Hard Sphere Model .....	72

2.4.2	The Inverse Power Law Model .....	74
2.4.3	Maxwell Model.....	76
2.4.4	Variable Hard Sphere (VHS) Model.....	77
2.4.5	Variable Soft Sphere (VSS) Model .....	80
2.4.6	Generalized Hard Sphere (GHS) Model .....	85
2.4.7	Generalized Soft Sphere (GSS) Model .....	87
2.5	The Eight Velocity Gas Model.....	88
2.6	Boltzmann Equation.....	92
2.7	Collision Integral and The Total Number of Collisions .....	98
2.8	Evaluation of Collision Integrals .....	100
2.9	The Maxwell Transport Equation – The Moment Equation .....	104
2.10	Maxwell Distribution.....	106
2.11	Equilibrium State of Gases .....	112
2.11.1	Some Peculiar Speeds of Gas .....	112
2.11.2	Molecular Collision Frequency and The Mean Free Path ....	115
2.11.3	The Mean Value of Collision Quantities .....	120
2.11.4	The Reference Diameter of The VSS Model and The VHS Model.....	123
2.12	Gas Mixture .....	124
2.12.1	The Macroscopic Properties.....	124
2.12.2	The Boltzmann Equations.....	126
2.12.3	Number of Collisions, Collision Frequency and Mean Free Path.....	126
2.12.4	Collision Frequency of a Molecule of species A with Molecules of Species B in Gas Mixture of VSS (or VHS) Molecules .....	127
	References .....	128
<b>3</b>	<b>Interaction of Molecules with Solid Surface</b> .....	<b>131</b>
3.1	Introduction.....	131
3.2	Specular and Diffuse Reflection .....	132
3.3	The Reciprocity Principle .....	140
3.4	The CLL Gas Surface Interaction Model.....	142
	References .....	158

---

<b>4</b>	<b>Free Molecular Flow</b>	<b>159</b>
4.1	The Number Flux and The Momentum Flux of Molecules in Gases .....	160
4.2	The Aerodynamic Forces Acted on Bodies .....	163
4.3	Heat Transfer to Surface Element.....	170
4.4	Free Molecular Effusion and Thermal Transpiration .....	174
4.5	Couette Flow and Heat Transfer between Plane Plates .....	177
4.6	The general solutions, unsteady flow.....	182
	References .....	190
<b>5</b>	<b>Continuum Models</b>	<b>191</b>
5.1	Introduction.....	191
5.2	Basic Equations.....	192
5.2.1	Equations of Mass, Momentum and Energy Conservation..	192
5.2.2	Chapman-Enskog Expansion.....	193
5.2.3	Euler Equation .....	194
5.2.4	Navier-Stokes Equations .....	194
5.2.5	Burnett Equations .....	196
5.2.6	Grad's Thirteen Moment Equations .....	201
5.2.7	The Asymptotic Theory for Small Knudsen Numbers .....	203
5.3	Slip Boundary Conditions.....	204
5.3.1	The Simple Derivation.....	204
5.3.2	The Conservation of Momentum and Energy Fluxes in The Knudsen Layer .....	206
5.3.3	The Derivation of The Slip Velocity Formula .....	207
5.3.4	The Derivation of The Temperature Jump Expression .....	209
5.3.5	The Extension to Cases of Multi-component Gases and Non-equilibrium Flows .....	212
5.4	The Solution of Some Simple Problems .....	212
5.4.1	Couette Flow.....	213
5.4.2	The Poiseuille Flow .....	215
5.4.3	The Rayleigh Problem .....	218
5.5	Thermal Creep and Thermophoresis .....	220
5.6	Second Order Slip-jump Conditions .....	227

---

References .....	228
<b>6 Transitional Regime</b>	<b>231</b>
6.1 General Overview.....	231
6.2 Linearized Boltzmann Equation.....	233
6.3 The Moment Method.....	239
6.4 Model Equations.....	247
6.5 The Finite Difference Method.....	255
6.6 Discrete Ordinate Method.....	257
6.7 Integral Methods.....	263
6.8 Direct Simulation Methods.....	264
References .....	269
<b>7 Direct Simulation Monte-Carlo (DSMC) Method</b>	<b>275</b>
7.1 Introduction.....	275
7.2 Sampling of Collisions.....	278
7.3 Example of Solution of Problem by The DSMC Method.....	281
7.4 The Excitation and Relaxation of The Internal Energies .....	288
7.4.1 Introduction of Phenomenological Models .....	288
7.4.2 Implementation of Larsen-Borgnakke Model.....	289
7.4.3 Cases of Distributions with Singularities, Generalized Acceptance-rejection Method.....	293
7.4.4 Larsen- Borgnakke Method for Discrete Energy Levels .....	295
7.4.5 Relaxation Collision Number and Vibrational Exchange Probability.....	297
7.5 Simulation of Chemical Reactions.....	299
7.5.1 Chemical Reaction Rate Coefficient .....	299
7.5.2 Phenomenological Chemical Reaction Model of Bird .....	300
7.5.3 A Sterically Dependent Chemical Reaction Model.....	302
7.6 Computation of Comp licated Flow Fields .....	310
References .....	313
<b>8 Microscale Slow Gas Flows , Information Preservation Method</b>	<b>317</b>
8.1 Introduction.....	317

---

8.2	Methods for Solving The Rarefied Gas Flows in MEMS .....	321
8.3	Information Preservation (IP) Method.....	326
8.3.1	The Description of The Method.....	326
8.3.2	The Validation of The Method.....	329
8.3.3	Program Demonstrating The Method.....	332
8.4	Unidirectional Flows.....	333
8.5	The Microchannel Flow Problem.....	338
8.6	Thin Film Air Bearing Problem.....	348
8.7	Use of Degenerated Reynolds Equation in Channel Flow .....	355
8.8	Some Actual Problems and Concluding Remarks .....	360
	References .....	363
<b>Appendix I Gas Properties</b>		<b>367</b>
	References .....	368
<b>Appendix II Some Integrals</b>		<b>369</b>
II.1	The gamma Function and Error Function .....	369
II.2	Some Definite Integrals .....	370
II.3	The beta Function.....	373
	References .....	374
<b>Appendix III Sampling from a Prescribed Distribution</b>		<b>375</b>
III.1	Inversion of Cumulative Distribution Function .....	375
III.2	Acceptance-rejection Method.....	378
III.3	Generalized Acceptance-rejection Method.....	378
	References .....	381
<b>Appendix IV Program of The Couette Flow</b>		<b>383</b>
<b>Subject Index</b>		<b>399</b>