
Contents

1	Introduction	1
1.1	Fundamental Processes in Fluid Dynamics and Their Coupling	2
1.2	Historical Development	3
1.3	The Contents of the Book	6

Part I Vorticity Dynamics

2	Fundamental Processes in Fluid Motion	13
2.1	Basic Kinematics	13
2.1.1	Descriptions and Visualizations of Fluid Motion	13
2.1.2	Deformation Kinematics, Vorticity and Dilatation	18
2.1.3	The Rate of Change of Material Integrals	22
2.2	Fundamental Equations of Newtonian Fluid Motion	25
2.2.1	Mass Conservation	25
2.2.2	Balance of Momentum and Angular Momentum	26
2.2.3	Energy Balance, Dissipation, and Entropy	28
2.2.4	Boundary Conditions, Fluid-Dynamic Force and Moment	30
2.2.5	Effectively Inviscid Flow and Surface of Discontinuity	33
2.3	Intrinsic Decompositions of Vector Fields	36
2.3.1	Functionally Orthogonal Decomposition	36
2.3.2	Integral Expression of Decomposed Vector Fields	40
2.3.3	Monge–Clebsch decomposition	43
2.3.4	Helical–Wave Decomposition	44
2.3.5	Tensor Potentials	47
2.4	Splitting and Coupling of Fundamental Processes	48
2.4.1	Triple Decomposition of Strain Rate and Velocity Gradient	49

2.4.2	Triple Decomposition of Stress Tensor and Dissipation	52
2.4.3	Internal and Boundary Coupling of Fundamental Processes	55
2.4.4	Incompressible Potential Flow	59
	Summary	63
3	Vorticity Kinematics	67
3.1	Physical Interpretation of Vorticity	67
3.2	Vorticity Integrals and Far-Field Asymptotics	71
3.2.1	Integral Theorems	71
3.2.2	Biot–Savart Formula	78
3.2.3	Far-Field Velocity Asymptotics	83
3.3	Lamb Vector and Helicity	85
3.3.1	Complex Lamellar, Beltrami, and Generalized Beltrami Flows	86
3.3.2	Lamb Vector Integrals, Helicity, and Vortex Filament Topology	90
3.4	Vortical Impulse and Kinetic Energy	94
3.4.1	Vortical Impulse and Angular Impulse	94
3.4.2	Hydrodynamic Kinetic Energy	97
3.5	Vorticity Evolution	100
3.5.1	Vorticity Evolution in Physical and Reference Spaces	100
3.5.2	Evolution of Vorticity Integrals	103
3.5.3	Enstrophy and Vorticity Line Stretching	105
3.6	Circulation-Preserving Flows	109
3.6.1	Local and Integral Conservation Theorems	109
3.6.2	Bernoulli Integrals	113
3.6.3	Hamiltonian Formalism	117
3.6.4	Relabeling Symmetry and Energy Extremum	120
3.6.5	Viscous Circulation-Preserving Flow	125
	Summary	127
4	Fundamentals of Vorticity Dynamics	131
4.1	Vorticity Diffusion Vector	131
4.1.1	Nonconservative Body Force in Magnetohydrodynamics	131
4.1.2	Baroclinicity	134
4.1.3	Viscosity Diffusion, Dissipation, and Creation at Boundaries	138
4.1.4	Unidirectional and Quasiparallel Shear Flows	144

4.2	Vorticity Field at Small Reynolds Numbers	150
4.2.1	Stokes Approximation of Flow Over Sphere	150
4.2.2	Oseen Approximation of Flow Over Sphere	153
4.2.3	Separated Vortex and Vortical Wake	155
4.2.4	Regular Perturbation	159
4.3	Vorticity Dynamics in Boundary Layers	161
4.3.1	Vorticity and Lamb Vector in Solid-Wall Boundary Layer	162
4.3.2	Vorticity Dynamics in Free-Surface Boundary Layer . . .	168
4.4	Vortex Sheet Dynamics	172
4.4.1	Basic Properties	173
4.4.2	Kutta Condition	178
4.4.3	Self-Induced Motion	179
4.4.4	Vortex Sheet Transport Equation	183
4.5	Vorticity-Based Formulation of Viscous Flow Problem	185
4.5.1	Kinematical Well-Posedness	187
4.5.2	Boundary Vorticity–Pressure Coupling	190
4.5.3	A Locally Decoupled Differential Formulation	191
4.5.4	An Exact Fully Decoupled Formulation	197
	Summary	199
5	Vorticity Dynamics in Flow Separation	201
5.1	Flow Separation and Boundary-Layer Separation	201
5.2	Three-Dimensional Steady Flow Separation	204
5.2.1	Near-Wall Flow in Terms of On-Wall Signatures	205
5.2.2	Local Separation Criteria	210
5.2.3	Slope of Separation Stream Surface	213
5.2.4	A Special Result on Curved Surface	215
5.3	Steady Boundary Layer Separation	216
5.3.1	Goldstein’s Singularity and Triple-Deck Structure	218
5.3.2	Triple-Deck Equations and Interactive Vorticity Generation	221
5.3.3	Boundary-Layer Separation in Two Dimensions	227
5.3.4	Boundary-Layer Separation in Three Dimensions	229
5.4	Unsteady Separation	234
5.4.1	Physical Phenomena of Unsteady Boundary-Layer Separation	235
5.4.2	Lagrangian Theory of Unsteady Boundary Layer Separation	240
5.4.3	Unsteady Flow Separation	246
	Summary	251

Part II Vortex Dynamics

6	Typical Vortex Solutions	255
6.1	Governing Equations.....	255
6.2	Axisymmetric Columnar Vortices.....	260
6.2.1	Stretch-Free Columnar Vortices.....	260
6.2.2	Viscous Vortices with Axial Stretching.....	263
6.2.3	Conical Similarity Swirling Vortices.....	268
6.3	Circular Vortex Rings.....	272
6.3.1	General Formulation and Induced Velocity.....	272
6.3.2	Fraenkel–Norbury Family and Hill Spherical Vortex....	277
6.3.3	Thin-Cored Pure Vortex Ring: Direct Method.....	281
6.3.4	Thin-Cored Swirling Vortex Rings: Energy Method....	283
6.4	Exact Strained Vortex Solutions.....	284
6.4.1	Strained Elliptic Vortex Patches.....	285
6.4.2	Vortex Dipoles.....	289
6.4.3	Vortex Arrays.....	291
6.5	Asymptotic Strained Vortex Solutions.....	295
6.5.1	Matched Asymptotic Expansion and Canonical Equations.....	296
6.5.2	Strained Solution in Distant Vortex Dipole.....	303
6.5.3	Vortex in Triaxial Strain Field.....	306
6.6	On the Definition of Vortex.....	310
6.6.1	Existing Criteria.....	310
6.6.2	An Analytical Comparison of the Criteria.....	314
6.6.3	Test Examples and Discussion.....	316
	Summary.....	320
7	Separated Vortex Flows	323
7.1	Topological Theory of Separated Flows.....	323
7.1.1	Fixed Points and Closed Orbits of a Dynamic System.....	324
7.1.2	Closed and Open Separations.....	327
7.1.3	Fixed-Point Index and Topology of Separated Flows.....	330
7.1.4	Structural Stability and Bifurcation of Separated Flows.....	332
7.2	Steady Separated Bubble Flows in Euler Limit.....	339
7.2.1	Prandtl–Batchelor Theorem.....	340
7.2.2	Plane Prandtl–Batchelor Flows.....	346
7.2.3	Steady Global Wake in Euler Limit.....	350
7.3	Steady Free Vortex-Layer Separated Flow.....	352
7.3.1	Slender Approximation of Free Vortex Sheet.....	353

7.3.2	Vortex Sheets Shed from Slender Wing	359
7.3.3	Stability of Vortex Pairs Over Slender Conical Body . . .	361
7.4	Unsteady Bluff-Body Separated Flow	366
7.4.1	Basic Flow Phenomena	367
7.4.2	Formation of Vortex Shedding	372
7.4.3	A Dynamic Model of the (St, C_D, Re) Relationship	376
	Summary	381
8	Core Structure, Vortex Filament, and Vortex System	383
8.1	Vortex Formation and Core Structure	383
8.1.1	Vortex Formation by Vortex-Layer Rolling Up	384
8.1.2	Quasicylindrical Vortex Core	387
8.1.3	Core Structure of Typical Vortices	390
8.1.4	Vortex Core Dynamics	395
8.2	Dynamics of Three-Dimensional Vortex Filament	399
8.2.1	Local Induction Approximation	401
8.2.2	Vortex Filament with Finite Core and Stretching	407
8.2.3	Nonlocal Effects of Self-Stretch and Background Flow	413
8.3	Motion and Interaction of Multiple Vortices	418
8.3.1	Two-Dimensional Point-Vortex System	418
8.3.2	Vortex Patches	424
8.3.3	Vortex Reconnection	431
8.4	Vortex–Boundary Interactions	434
8.4.1	Interaction of Vortex with a Body	435
8.4.2	Interaction of Vortex with Fluid Interface	441
	Summary	446

Part III Vortical Flow Instability, Transition and Turbulence

9	Vortical-Flow Stability and Vortex Breakdown	451
9.1	Fundamentals of Hydrodynamic Stability	451
9.1.1	Normal-Mode Linear Stability	453
9.1.2	Linear Instability with Non-normal Operator	458
9.1.3	Energy Method and Inviscid Arnold Theory	462
9.1.4	Linearized Disturbance Lamb Vector and the Physics of Instability	467
9.2	Shear-Flow Instability	469
9.2.1	Instability of Parallel Shear Flow	469
9.2.2	Instability of free shear flow	472
9.2.3	Instability of Boundary Layer	475
9.2.4	Non-Normal Effects in Shear-Flow Instability	477

9.3	Instability of Axisymmetric Columnar Vortices	480
9.3.1	Stability of Pure Vortices	480
9.3.2	Temporal Instability of Swirling Flow	481
9.3.3	Absolute and Convective Instability of Swirling Flow	485
9.3.4	Non-Modal Instability of Vortices	488
9.4	Instabilities of Strained Vortices	492
9.4.1	Elliptical Instability	493
9.4.2	A Columnar Vortex in a Strained Field	496
9.4.3	Instability of a Vortex Pair	499
9.5	Vortex Breakdown	502
9.5.1	Vorticity-Dynamics Mechanisms of Vortex Breakdown	504
9.5.2	Onset of Vortex Breakdown: Fold Catastrophe Theory	506
9.5.3	Vortex Breakdown Development: AI/CI Analysis	511
	Summary	515

10 Vortical Structures in Transitional and Turbulent

	Shear Flows	519
10.1	Coherent Structures	520
10.1.1	Coherent Structures and Vortices	520
10.1.2	Scaling Problem in Coherent Structure	522
10.1.3	Coherent Structure and Wave	524
10.2	Vortical Structures in Free Shear Flows	526
10.2.1	Instability of Free Shear Layers and Formation of Spanwise Vortices	526
10.2.2	The Secondary Instability and Formation of Streamwise Vortices	530
10.2.3	Vortex Interaction and Small-Scale Transition	532
10.3	Vortical Structures in Wall-Bounded Shear Layers	535
10.3.1	Tollmien–Schlichting Instability and Formation of Initial Streaks	536
10.3.2	Secondary Instability and Self-Sustaining Cycle of Structure Regeneration	539
10.3.3	Small-Scale Transition in Boundary Layers	541
10.3.4	A General Description of Turbulent Boundary Layer Structures	545
10.3.5	Streamwise Vortices and By-Pass Transition	548
10.4	Some Theoretical Aspects in Studying Coherent Structures	550
10.4.1	On the Reynolds Decomposition	551
10.4.2	On Vorticity Transport Equations	556
10.4.3	Vortex Core Dynamics and Polarized Vorticity Dynamics	559

10.5	Two Basic Processes in Turbulence	561
10.5.1	Coherence Production – the First Process	562
10.5.2	Cascading – the Second Process	566
10.5.3	Flow Chart of Coherent Energy and General Strategy of Turbulence Control.....	567
10.6	Vortical Structures in Other Shear Flows	573
10.6.1	Vortical Structures in Plane Complex Turbulent Shear Flows	573
10.6.2	Vortical Structures in Nonplanar Shear Flows	577
10.6.3	Vortical Flow Shed from Bluff Bodies	580
	Summary	583

Part IV Special Topics

11	Vortical Aerodynamic Force and Moment.....	587
11.1	Introduction	587
11.1.1	The Need for “Nonstandard” Theories	588
11.1.2	The Legacy of Pioneering Aerodynamicist	590
11.1.3	Exact Integral Theories with Local Dynamics	593
11.2	Projection Theory	594
11.2.1	General Formulation	595
11.2.2	Diagnosis of Pressure Force Constituents	597
11.3	Vorticity Moments and Classic Aerodynamics	599
11.3.1	General Formulation	600
11.3.2	Force, Moment, and Vortex Loop Evolution.....	603
11.3.3	Force and Moment on Unsteady Lifting Surface	606
11.4	Boundary Vorticity-Flux Theory	608
11.4.1	General Formulation	608
11.4.2	Airfoil Flow Diagnosis	611
11.4.3	Wing-Body Combination Flow Diagnosis	615
11.5	A DMT-Based Arbitrary-Domain Theory.....	617
11.5.1	General Formulation	617
11.5.2	Multiple Mechanisms Behind Aerodynamic Forces	621
11.5.3	Vortex Force and Wake Integrals in Steady Flow	627
11.5.4	Further Applications.....	633
	Summary	639
12	Vorticity and Vortices in Geophysical Flows	641
12.1	Governing Equations and Approximations	642
12.1.1	Effects of Frame Rotation and Density Stratification ...	642
12.1.2	Boussinesq Approximation.....	646
12.1.3	The Taylor–Proudman Theorem.....	648
12.1.4	Shallow-Water Approximation	649

12.2	Potential Vorticity	652
12.2.1	Barotropic (Rossby) Potential Vorticity	653
12.2.2	Geostrophic and Quasigeostrophic Flows	654
12.2.3	Rossby Wave	656
12.2.4	Baroclinic (Ertel) Potential Vorticity.....	659
12.3	Quasigeostrophic Evolution of Vorticity and Vortices	664
12.3.1	The Evolution of Two-Dimensional Vorticity Gradient	665
12.3.2	The Structure and Evolution of Barotropic Vortices....	670
12.3.3	The Structure of Baroclinic Vortices	676
12.3.4	The Propagation of Tropical Cyclones.....	680
	Summary	690
A	Vectors, Tensors, and Their Operations	693
A.1	Vectors and Tensors	693
A.1.1	Scalars and Vectors	693
A.1.2	Tensors	694
A.1.3	Unit Tensor and Permutation Tensor	696
A.2	Integral Theorems and Derivative Moment Transformation ...	698
A.2.1	Generalized Gauss Theorem and Stokes Theorem	698
A.2.2	Derivative Moment Transformation on Volume	700
A.2.3	Derivative Moment Transformation on Surface	701
A.2.4	Special Issues in Two Dimensions	703
A.3	Curvilinear Frames on Lines and Surfaces	705
A.3.1	Intrinsic Line Frame	705
A.3.2	Intrinsic operation with surface frame	707
A.4	Applications in Lagrangian Description	716
A.4.1	Deformation Gradient Tensor and its Inverse.....	716
A.4.2	Images of Physical Vectors in Reference Space	717
	References	721
	Index	767