Contents

Abbreviations

Arithmetics

a, ..., z	variables, a, ..., $z \in \mathbb{R}$
$\underline{a}$, ..., $\underline{z}$	vectors
$\underline{A}$, ..., $\underline{Z}$	matrices
$\sum$	addition operator of real-valued variables
$\prod$	multiplication operator of real-valued variables
$\bigoplus$	$l_\alpha r_\alpha$-addition operator of fuzzy variables
ln	natural logarithm

Analysis

$(... ; ...)^T$	elements of a column matrix
$(... ; ...)$	open interval
$[... ; ...]$	closed interval
$\lvert ... \rvert$	absolute value
lim	limes, limit
∞	infinity
d	differentiation
∂	partial differentiation
$\int$	integration
$\rightarrow$	mapping

Set Theory

$\mathbf{A}$, ..., $\mathbf{Z}$	fundamental sets
A, ..., Z	sets
$\mathbb{N}$	natural numbers
$\mathbb{R}$	real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\{\ldots ; \ldots\}$	set of ..., elements of a set
$\in$	element of
$\subseteq$	subset of
$\cap$	intersection
$\cup$	union
$\emptyset$	empty set

Logic

$\wedge$	conjunction, logical *and*
$\vee$	alternative, logical *or*
$\mid$	for which the following holds
$\forall$	universal quantifier, for all
$\exists$	existential quantifier, there exists

Fuzzy Set Theory

$\sim$	fuzziness
$\tilde{a}$, ..., $\tilde{z}$	fuzzy variables
x_l	peak point of $\tilde{x}$
$S_{\tilde{x}}$	support of $\tilde{x}$
$\mu_{\tilde{x}}(x)$	membership function of $\tilde{x}$
α_i	ith α-level
X_{α_i}	α-level set of $\tilde{x}$ for $\alpha = \alpha_i$
n	number of discrete α-levels
$\Delta x_{\alpha_i l}$	l_α-increment of $\tilde{x}$ for $\alpha = \alpha_i$
$\Delta x_{\alpha_i r}$	r_α-increment of $\tilde{x}$ for $\alpha = \alpha_i$
Δx_i	ith element of the $l_\alpha r_\alpha$-increment representation of $\tilde{x}$
$L_{\alpha_{i,i+1}}(\cdot)$	sub-function of $\mu_{\tilde{x}}(x)$ between $\alpha_i \leqslant \alpha < \alpha_{i+1}$ (left)
$R_{\alpha_{i,i+1}}(\cdot)$	sub-function of $\mu_{\tilde{x}}(x)$ between $\alpha_i \leqslant \alpha < \alpha_{i+1}$ (right)
$\hat{\tilde{z}}$	best possible approximation of $\tilde{z}$

$d_F(\tilde{a};\tilde{b})$	distance between two fuzzy variables $\tilde{a}$ and $\tilde{b}$
$d_H(A;B)$	Hausdorff distance between two sets A and B
$d_E(a;b)$	Euclidean distance between two real-valued variables a and b
sup	supremum operator
inf	infimum operator
max	maximum operator
min	minimum operator

Fuzzy Probabilistics

A, ..., Z	random variables
Ã, ..., Z̃	fuzzy random variables
ω	random elementary event
Ω	space of the random elementary events
$P(\square)$	probability of occurence of $\square$
$\tilde{P}(\square)$	fuzzy probability of occurence of $\square$
$F_{\mathrm{X}}(x)$	probability distribution function of X
$f_{\mathrm{X}}(x)$	probability density function of X
$\tilde{F}_{\tilde{\mathrm{X}}}(x)$	fuzzy probability distribution function form I of $\tilde{\mathrm{X}}$
$F_{\tilde{\mathrm{X}}}(\tilde{x})$	fuzzy probability distribution function form II of $\tilde{\mathrm{X}}$
${}_{lr}F_{\tilde{\mathrm{X}}}(\tilde{x})$	fuzzy probability distribution function form II of $\tilde{\mathrm{X}}$ using $l_\alpha r_\alpha$-discretization
${}_{lr}f_{\tilde{\mathrm{X}}}(\tilde{x})$	fuzzy probability density function form II of $\tilde{\mathrm{X}}$ using $l_\alpha r_\alpha$-discretization

Artificial Neural Networks for Fuzzy Variables

I	input layer
H_i	ith hidden layer
O	output layer
n_I	number of neurons in the input layer
n_{H_i}	number of neurons in the ith hidden layer
n_O	number of neurons in the output layer
$\tilde{o}_i^{\square}$	fuzzy output value of the ith neuron of the layer $\square$
$\underline{\tilde{W}}_{ij}^{\square}$	weighting matrix of the jth neuron of the layer $\square$ referring to the ith neuron of the preceding layer
$\tilde{net}_i^{\square}$	fuzzy netto input of the ith neuron of the layer $\square$
$\tilde{f}_A(\cdot)$	fuzzy activation function

$f_A(\cdot)$	deterministic activation function
$\tilde{f}_O(\cdot)$	fuzzy output function
$f_O(\cdot)$	deterministic output function
$\tilde{\theta}_i^{\square}$	fuzzy bias of the ith neuron of the layer $\square$
$\underline{\tilde{x}}_i$	ith fuzzy input vector
$\underline{\tilde{y}}_i$	ith fuzzy control vector

Analysis of Time Series comprised of Fuzzy Data

τ	time coordinate
$\mathbf{T}$	set of equidistant points in time
$(\tilde{x}_\tau)_{\tau\in\mathbf{T}}$	fuzzy time series
$\tilde{x}_\tau$	fuzzy variable at point in time τ
$\tilde{t}(\tau)$	fuzzy trend function
$\tilde{t}_\tau$	functional value of $\tilde{t}(\tau)$ at point in time τ
$\tilde{z}(\tau)$	fuzzy cycle function
$\tilde{z}_\tau$	functional value of $\tilde{z}(\tau)$ at point in time τ
$(\tilde{\mathrm{R}}_\tau)_{\tau\in\mathbf{T}}$	fuzzy random residual process
$\tilde{r}_\tau$	realization of $(\tilde{\mathrm{R}}_\tau)_{\tau\in\mathbf{T}}$ at point in time τ, fuzzy residual component
$t_j^*(\tau)$	trend function of the jth $l_\alpha r_\alpha$-increment
λ_j	frequency of the cyclic variation of the jth $l_\alpha r_\alpha$-increment
$z_j^*(\tau)$	cycle function of the jth $l_\alpha r_\alpha$-increment
$\bar{\tilde{x}}$	fuzzy mean value of $(\tilde{x}_\tau)_{\tau\in\mathbf{T}}$
${}_{lr}\underline{s}^2_{\tilde{x}_\tau}$	$l_\alpha r_\alpha$-variance of $(\tilde{x}_\tau)_{\tau\in\mathbf{T}}$
${}_{lr}\underline{s}_{\tilde{x}_\tau}$	$l_\alpha r_\alpha$-standard deviation of $(\tilde{x}_\tau)_{\tau\in\mathbf{T}}$
${}_{lr}\underline{\hat{K}}_{\tilde{x}_\tau}(\Delta\tau)$	$l_\alpha r_\alpha$-covariance function of $(\tilde{x}_\tau)_{\tau\in\mathbf{T}}$
${}_{lr}\underline{\hat{R}}_{\tilde{x}_\tau}(\Delta\tau)$	$l_\alpha r_\alpha$-correlation function of $(\tilde{x}_\tau)_{\tau\in\mathbf{T}}$
L	linear filter for fuzzy time series
c_i	ith filter coefficient
D^p	fuzzy difference filter of the order p
L_e	extended linear filter for fuzzy time series
$\underline{C}_i$	ith coefficient matrix
D_e^p	extended fuzzy difference filter of the order p
$(\tilde{\mathrm{X}}_\tau)_{\tau\in\mathbf{T}}$	fuzzy random process
$\tilde{\mathrm{X}}_\tau$	fuzzy random variable at point in time τ
$E[\tilde{\mathrm{X}}_\tau]$, $\tilde{m}_{\tilde{\mathrm{X}}_\tau}$	fuzzy expected value of $\tilde{\mathrm{X}}_\tau$
${}_{lr}Var[\tilde{\mathrm{X}}_\tau]$, ${}_{lr}\underline{\sigma}^2_{\tilde{\mathrm{X}}_\tau}$	$l_\alpha r_\alpha$-variance of $\tilde{\mathrm{X}}_\tau$

${}_{lr}\underline{\sigma}_{\tilde{\mathrm{X}}_\tau}$	$l_\alpha r_\alpha$-standard deviation of of $\tilde{\mathrm{X}}_\tau$
${}_{lr}\underline{K}_{\tilde{\mathrm{X}}_\tau}(\tau_a, \tau_b)$	$l_\alpha r_\alpha$-covariance function of $(\tilde{\mathrm{X}}_\tau)_{\tau\in\mathbf{T}}$
${}_{lr}\underline{R}_{\tilde{\mathrm{X}}_\tau}(\tau_a, \tau_b)$	$l_\alpha r_\alpha$-correlation function of $(\tilde{\mathrm{X}}_\tau)_{\tau\in\mathbf{T}}$
AR	autoregressive
MA	moving average
ARMA	autoregressive moving average
$(\tilde{\mathcal{E}}_\tau)_{\tau\in\mathbf{T}}$	fuzzy white-noise prozess
ε_τ	realization of $(\tilde{\mathcal{E}}_\tau)_{\tau\in\mathbf{T}}$ at point in time τ
p	order of fuzzy AR processes
q	order of fuzzy MA processes
$\underline{A}_i$	ith coefficient matrix of fuzzy AR processes
$\underline{B}_i$	ith coefficient matrix of fuzzy MA processes
$\underline{C}_i$	ith coefficient matrix of inverted fuzzy MA processes
$\underline{\Theta}_i$	ith parameter matrix of the WILSON-algorithm for fuzzy MA processes
$\underline{\Delta}_i$	ith correction matrix of the WILSON-algorithm for fuzzy MA processes

Forecasting of Time Series comprised of Fuzzy Data

N	number of observed fuzzy variables of a fuzzy time series
h	forecasting step size
$(\vec{\tilde{\mathrm{X}}}_\tau)_{\tau\in\mathbf{T}}$	fuzzy random forecasting process
$\vec{\tilde{\mathrm{X}}}_\tau$	conditional fuzzy random variable at point in time τ
$\vec{\tilde{x}}_{N+h}$	realization of $\vec{\tilde{\mathrm{X}}}_\tau$ at point in time $\tau = N + h$
$\mathring{\tilde{x}}_{N+h}$	optimum forecast at point in time $\tau = N + h$
$\tilde{x}^{\kappa}_{N+h}$	fuzzy forecast interval with confidence level κ
$\tilde{x}^{*}_{N+h}$	conditional fuzzy forecast interval
$P^{*}_{X_{N+h}}$	conditional probability for $\hat{\tilde{\mathrm{X}}}_{N+h} \subseteq \tilde{x}^{*}_{N+h}$