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# Abbreviations

## Arithmetics

$a, \dots, z$	variables, $a, \dots, z \in \mathbb{R}$
$\underline{a}, \dots, \underline{z}$	vectors
$\underline{A}, \dots, \underline{Z}$	matrices
$\Sigma$	addition operator of real-valued variables
$\Pi$	multiplication operator of real-valued variables
$\oplus$	$l_\alpha r_\alpha$ -addition operator of fuzzy variables
$\ln$	natural logarithm

## Analysis

$(\dots; \dots)^T$	elements of a column matrix
$(\dots; \dots)$	open interval
$[\dots; \dots]$	closed interval
$ \dots $	absolute value
$\lim$	limes, limit
$\infty$	infinity
$d$	differentiation
$\partial$	partial differentiation
$\int$	integration
$\rightarrow$	mapping

**Set Theory**

$\mathbf{A}, \dots, \mathbf{Z}$	fundamental sets
$A, \dots, Z$	sets
$\mathbb{N}$	natural numbers
$\mathbb{R}$	real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\{\dots; \dots\}$	set of ..., elements of a set
$\in$	element of
$\subseteq$	subset of
$\cap$	intersection
$\cup$	union
$\emptyset$	empty set

**Logic**

$\wedge$	conjunction, logical <i>and</i>
$\vee$	alternative, logical <i>or</i>
$ $	for which the following holds
$\forall$	universal quantifier, for all
$\exists$	existential quantifier, there exists

**Fuzzy Set Theory**

$\sim$	fuzziness
$\tilde{a}, \dots, \tilde{z}$	fuzzy variables
$x_l$	peak point of $\tilde{x}$
$S_{\tilde{x}}$	support of $\tilde{x}$
$\mu_{\tilde{x}}(x)$	membership function of $\tilde{x}$
$\alpha_i$	<i>i</i> th $\alpha$ -level
$X_{\alpha_i}$	$\alpha$ -level set of $\tilde{x}$ for $\alpha = \alpha_i$
$n$	number of discrete $\alpha$ -levels
$\Delta x_{\alpha_i l}$	$l_\alpha$ -increment of $\tilde{x}$ for $\alpha = \alpha_i$
$\Delta x_{\alpha_i r}$	$r_\alpha$ -increment of $\tilde{x}$ for $\alpha = \alpha_i$
$\Delta x_i$	<i>i</i> th element of the $l_\alpha r_\alpha$ -increment representation of $\tilde{x}$
$L_{\alpha_i, \alpha_{i+1}}(\cdot)$	sub-function of $\mu_{\tilde{x}}(x)$ between $\alpha_i \leq \alpha < \alpha_{i+1}$ (left)
$R_{\alpha_i, \alpha_{i+1}}(\cdot)$	sub-function of $\mu_{\tilde{x}}(x)$ between $\alpha_i \leq \alpha < \alpha_{i+1}$ (right)
$\hat{\tilde{z}}$	best possible approximation of $\tilde{z}$

$d_F(\tilde{a}; \tilde{b})$	distance between two fuzzy variables $\tilde{a}$ and $\tilde{b}$
$d_H(A; B)$	Hausdorff distance between two sets $A$ and $B$
$d_E(a; b)$	Euclidean distance between two real-valued variables $a$ and $b$
sup	supremum operator
inf	infimum operator
max	maximum operator
min	minimum operator

### Fuzzy Probabilistics

$A, \dots, Z$	random variables
$\tilde{A}, \dots, \tilde{Z}$	fuzzy random variables
$\omega$	random elementary event
$\Omega$	space of the random elementary events
$P(\square)$	probability of occurrence of $\square$
$\tilde{P}(\square)$	fuzzy probability of occurrence of $\square$
$F_X(x)$	probability distribution function of $X$
$f_X(x)$	probability density function of $X$
$\tilde{F}_{\tilde{X}}(x)$	fuzzy probability distribution function form I of $\tilde{X}$
$F_{\tilde{X}}(\tilde{x})$	fuzzy probability distribution function form II of $\tilde{X}$
${}_{lr}F_{\tilde{X}}(\tilde{x})$	fuzzy probability distribution function form II of $\tilde{X}$ using $l_\alpha r_\alpha$ -discretization
${}_{lr}f_{\tilde{X}}(\tilde{x})$	fuzzy probability density function form II of $\tilde{X}$ using $l_\alpha r_\alpha$ -discretization

### Artificial Neural Networks for Fuzzy Variables

$I$	input layer
$H_i$	$i$ th hidden layer
$O$	output layer
$n_I$	number of neurons in the input layer
$n_{H_i}$	number of neurons in the $i$ th hidden layer
$n_O$	number of neurons in the output layer
$\tilde{o}_i^\square$	fuzzy output value of the $i$ th neuron of the layer $\square$
$\underline{W}_{ij}^\square$	weighting matrix of the $j$ th neuron of the layer $\square$ referring to the $i$ th neuron of the preceding layer
$\tilde{net}_i^\square$	fuzzy netto input of the $i$ th neuron of the layer $\square$
$\tilde{f}_A(\cdot)$	fuzzy activation function

$f_A(\cdot)$	deterministic activation function
$\tilde{f}_O(\cdot)$	fuzzy output function
$f_O(\cdot)$	deterministic output function
$\tilde{\theta}_i^p$	fuzzy bias of the $i$ th neuron of the layer $p$
$\tilde{x}_i$	$i$ th fuzzy input vector
$\tilde{y}_i$	$i$ th fuzzy control vector

### Analysis of Time Series comprised of Fuzzy Data

$\tau$	time coordinate
$\mathbf{T}$	set of equidistant points in time
$(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$	fuzzy time series
$\tilde{x}_\tau$	fuzzy variable at point in time $\tau$
$\tilde{t}(\tau)$	fuzzy trend function
$\tilde{t}_\tau$	functional value of $\tilde{t}(\tau)$ at point in time $\tau$
$\tilde{z}(\tau)$	fuzzy cycle function
$\tilde{z}_\tau$	functional value of $\tilde{z}(\tau)$ at point in time $\tau$
$(\tilde{\mathbf{R}}_\tau)_{\tau \in \mathbf{T}}$	fuzzy random residual process
$\tilde{r}_\tau$	realization of $(\tilde{\mathbf{R}}_\tau)_{\tau \in \mathbf{T}}$ at point in time $\tau$ , fuzzy residual component
$t_j^*(\tau)$	trend function of the $j$ th $l_\alpha r_\alpha$ -increment
$\lambda_j$	frequency of the cyclic variation of the $j$ th $l_\alpha r_\alpha$ -increment
$z_j^*(\tau)$	cycle function of the $j$ th $l_\alpha r_\alpha$ -increment
$\tilde{x}$	fuzzy mean value of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\sigma}_{\tilde{x}_\tau}^2$	$l_\alpha r_\alpha$ -variance of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\sigma}_{\tilde{x}_\tau}$	$l_\alpha r_\alpha$ -standard deviation of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\tilde{K}}_{\tilde{x}_\tau}(\Delta\tau)$	$l_\alpha r_\alpha$ -covariance function of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{\tilde{R}}_{\tilde{x}_\tau}(\Delta\tau)$	$l_\alpha r_\alpha$ -correlation function of $(\tilde{x}_\tau)_{\tau \in \mathbf{T}}$
$L$	linear filter for fuzzy time series
$c_i$	$i$ th filter coefficient
$D^p$	fuzzy difference filter of the order $p$
$L_e$	extended linear filter for fuzzy time series
$\underline{C}_i$	$i$ th coefficient matrix
$D_e^p$	extended fuzzy difference filter of the order $p$
$(\tilde{\mathbf{X}}_\tau)_{\tau \in \mathbf{T}}$	fuzzy random process
$\tilde{\mathbf{X}}_\tau$	fuzzy random variable at point in time $\tau$
$E[\tilde{\mathbf{X}}_\tau], \tilde{m}_{\tilde{\mathbf{X}}_\tau}$	fuzzy expected value of $\tilde{\mathbf{X}}_\tau$
$l_r Var[\tilde{\mathbf{X}}_\tau], l_r \underline{\sigma}_{\tilde{\mathbf{X}}_\tau}^2$	$l_\alpha r_\alpha$ -variance of $\tilde{\mathbf{X}}_\tau$

$l_r \underline{\sigma}_{\tilde{X}_\tau}$	$l_\alpha r_\alpha$ -standard deviation of of $\tilde{X}_\tau$
$l_r \underline{K}_{\tilde{X}_\tau}(\tau_a, \tau_b)$	$l_\alpha r_\alpha$ -covariance function of $(\tilde{X}_\tau)_{\tau \in \mathbf{T}}$
$l_r \underline{R}_{\tilde{X}_\tau}(\tau_a, \tau_b)$	$l_\alpha r_\alpha$ -correlation function of $(\tilde{X}_\tau)_{\tau \in \mathbf{T}}$
AR	autoregressive
MA	moving average
ARMA	autoregressive moving average
$(\tilde{\mathcal{E}}_\tau)_{\tau \in \mathbf{T}}$	fuzzy white-noise process
$\varepsilon_\tau$	realization of $(\tilde{\mathcal{E}}_\tau)_{\tau \in \mathbf{T}}$ at point in time $\tau$
$p$	order of fuzzy AR processes
$q$	order of fuzzy MA processes
$\underline{A}_i$	$i$ th coefficient matrix of fuzzy AR processes
$\underline{B}_i$	$i$ th coefficient matrix of fuzzy MA processes
$\underline{C}_i$	$i$ th coefficient matrix of inverted fuzzy MA processes
$\underline{\Theta}_i$	$i$ th parameter matrix of the WILSON-algorithm for fuzzy MA processes
$\underline{\Delta}_i$	$i$ th correction matrix of the WILSON-algorithm for fuzzy MA processes

### Forecasting of Time Series comprised of Fuzzy Data

$N$	number of observed fuzzy variables of a fuzzy time series
$h$	forecasting step size
$(\vec{X}_\tau)_{\tau \in \mathbf{T}}$	fuzzy random forecasting process
$\vec{X}_\tau$	conditional fuzzy random variable at point in time $\tau$
$\vec{x}_{N+h}$	realization of $\vec{X}_\tau$ at point in time $\tau = N + h$
$\hat{x}_{N+h}$	optimum forecast at point in time $\tau = N + h$
$\tilde{x}_{N+h}^\kappa$	fuzzy forecast interval with confidence level $\kappa$
$\tilde{x}_{N+h}^*$	conditional fuzzy forecast interval
$P_{X_{N+h}}^*$	conditional probability for $\hat{X}_{N+h} \subseteq \tilde{x}_{N+h}^*$