

Table of Contents

Preface	iv
List of Symbols	viii
List of Figures	ix
List of Tables	xi

Part I Introduction

1 Introduction	3
1.1 Major Methodologies in Artificial Intelligence	3
1.2 Basic Academic Ideas	5
1.3 Some Related Concepts	7
1.4 Many-Valued Logic and Lattice-Valued Logic	12
1.5 Uncertainty Inference	16
1.5.1 Probability-Based Uncertainty Reasoning	16
1.5.2 Fuzzy Set Based Uncertainty Reasoning	19
1.5.3 Non-Monotonic Logic Based Uncertainty Reasoning...	20
1.6 Automated Reasoning in Many-Valued Logic	22

Part II Lattice Implication Algebras

2 Concepts and Properties	27
2.1 Lattice Implication Algebras	27
2.1.1 Concepts and Examples	27
2.1.2 Basic Properties	32
2.2 Lattice H Implication Algebras	47
2.3 Lattice Properties	50
2.4 Homomorphisms	53

3	Filters	59
3.1	Filters and Implicative Filters	59
3.2	Generated Filters	65
3.3	Positive Implicative Filters and Associative Filters	69
3.4	Prime Filters and Ultra-Filters	72
3.5	<i>I</i> -Filters, Involution Filters and Obstinate Filters	77
3.6	Fuzzy Filters	80
4	<i>LI</i>-Ideals	85
4.1	<i>LI</i> -Ideals	85
4.2	Fuzzy <i>LI</i> -Ideals	88
4.3	Normal Fuzzy <i>LI</i> -Ideals	90
4.4	Intuitionistic Fuzzy <i>LI</i> -Ideals	94
5	Homomorphisms and Representations	101
5.1	Congruence Relations	101
5.1.1	Congruence Relations Induced by Filters	101
5.1.2	Congruences Relations Induced by <i>LI</i> -ideals	111
5.1.3	Congruence Relations Induced by Fuzzy Filters	113
5.1.4	Congruence Relations Induced by Fuzzy <i>LI</i> -ideals	119
5.2	Proper Lattice Implication Algebras	122
5.3	Representations	127
6	Topological Structure of Filter Spaces	135
6.1	Filter Spaces	135
6.1.1	Basic Concepts	135
6.1.2	Topological Properties	138
6.2	Product Topology and Quotient Topology	141
6.3	Lattice Topology	144
6.4	Prime Spaces	145
7	Connections with Related Algebras	153
7.1	Lattice Implication Algebras and <i>BCK</i> -Algebras	153
7.2	Lattice Implication Algebras and <i>MV</i> -Algebras	157
7.3	Lattice Implication Algebras and Related Algebras	162
8	Related Issues	171
8.1	Category of Lattice Implication Algebras	171
8.2	Category of Fuzzy Lattice Implication Algebras	178
8.3	Fuzzy Power Sets	185
8.4	Adjoint Semigroups	192
8.5	Logical Properties	200

Part III Lattice-Valued Logic Systems

9	Lattice-Valued Propositional Logics	207
9.1	Lattice-Valued Propositional Logic $LP(X)$	207
9.1.1	Language	207
9.1.2	Semantics	207
9.1.3	Syntax	214
9.1.4	Examples	226
9.2	Gradational Lattice-Valued Propositional Logic L_{vpl}	227
9.2.1	Language	227
9.2.2	Rules of Inference	228
9.2.3	Semantics	233
9.2.4	Syntax	240
9.2.5	Satisfiability and Consistency	246
9.2.6	Deduction Theorem	249
9.2.7	Compactness	251
9.2.8	Examples	256
10	Lattice-Valued First-Order Logics	259
10.1	Lattice-Valued First-Order Logic $LF(X)$	259
10.1.1	Language	259
10.1.2	Interpretation	260
10.1.3	Semantics	261
10.1.4	Syntax	265
10.1.5	Properties of Model Theory	272
10.2	Gradational Lattice-Valued First-Order Logic L_{vfl}	278
10.2.1	Language	278
10.2.2	Interpretation	278
10.2.3	Semantics	280
10.2.4	Standardization of Formulae	290
10.2.5	Syntax	294
10.2.6	Soundness and Completeness	300
10.2.7	Satisfiability and Consistency	301
10.2.8	Deduction Theorem	302
10.2.9	Compactness	302
10.2.10	Examples	303
11	Uncertainty and Automated Reasoning	305
11.1	Uncertainty Reasoning Based on $LP(X)$	305
11.2	Uncertainty Reasoning Based on L_{vpl}	310

11.2.1	Another Kind of Interpretation of $X \models Y$	310
11.2.2	Basic Theory	311
11.2.3	Examples	319
11.2.4	Multi-Dimensional and Multiple Uncertainty Reasoning	322
	Models and Methods	322
	Semantical Interpretation and Syntactical Proof	324
11.3	α -Resolution Principle Based on $LP(X)$	328
11.3.1	α -Resolution Principle	328
11.3.2	Soundness and Completeness	333
11.4	α -Resolution Principle Based on $LF(X)$	349
11.4.1	Interpretation of Formulae	350
11.4.2	α -Resolution Principle	353
References		361
Index		389