

Contents

1.	Introduction	1
2.	Nonlinear Electrodynamics: Quantum Theory	3
2.1	Introduction to Proper-Time Methods	3
2.2	Polarization Tensor in External Fields	12
2.2.1	Derivation of $\Pi^{\mu\nu}$	14
2.2.2	Applications	20
2.3	Induced Electromagnetic Current	32
2.4	Schwinger's Equivalence Theorem and the Axial-Vector Anomaly	46
2.4.1	Equivalence Theorem	46
2.4.2	Axial-Vector Anomaly	57
3.	Nonlinear Electrodynamics: Effective-Action Approach	61
3.1	A First Look at Light Propagation	62
3.2	Light Cone Condition	65
3.2.1	Effective-Action Charge	69
3.2.2	Weak Electromagnetic Fields	70
3.2.3	Strong Magnetic Fields	71
3.2.4	Superstrong Magnetic Fields	74
3.3	Light Cone Condition for Various Vacua	79
3.3.1	"Unified Formula"	79
3.3.2	Light Cone Condition for Low-Energy Phenomena	83
3.3.3	Light Cone Conditions for High-Energy Phenomena	88
3.3.4	Measurability and Causality	89
3.4	Photon Splitting	93
3.5	QED Effective Action at Finite Temperature to One Loop	99
3.5.1	Imaginary-Time Formalism	101
3.5.2	Covariant Formulation	107
3.5.3	Effective Action	109
3.5.4	The \bar{A}_u Field	114
3.5.5	Temperature-Induced Pair Production at One Loop?	117

3.5.6	Discussion	118
3.6	QED Effective Action at Finite Temperature:	
	Two-Loop Dominance	119
3.6.1	Two-Loop Effective Action of QED	
	at Low Temperature	121
3.6.2	Light Propagation	131
3.6.3	Photon Splitting	145
3.6.4	Pair Production	150
3.6.5	Discussion	153
4.	QED in Two Spatial Dimensions	155
4.1	General Features of QED_{2+1}	156
4.1.1	Classical Properties	156
4.1.2	Quantum Theory	158
4.2	Parity-Invariant QED_{2+1}	161
4.3	The Effective Action for Irreducible QED_{2+1}	165
4.3.1	QED_{2+1} for Nonconstant Fields	174
5.	Scattering of Light by Light	181
5.1	Photon–Photon Scattering at Low Energies	181
5.2	High-Energy Photon Scattering	
	Close to the Forward Direction	184
Appendices	191
A	Units, Metric and the Gamma Matrices	191
B	Invariants and Spectral Representation	
	of the Maxwell Field Strength Tensor	193
B.1	Invariants	193
B.2	Spectral Representation of $F^{\mu\nu}$	195
B.3	Applications	197
C	More about the Commutator $\sigma^{\mu\nu}$ of Dirac Matrices	198
D	Elementary Calculations	204
D.1	ϵ expansions	204
D.2	Identities for Special Functions	204
D.3	Nonstandard Integrals	205
D.4	ϵ -Integration Techniques	205
D.5	$\Pi^{\mu\nu}$ for Strong Magnetic Fields	209
D.6	Generalized Gamma Function	
	and Hurwitz Zeta Function	213
D.7	Q Factor for Strong Magnetic Fields	215
D.8	Summation Techniques for Finite-Temperature Physics	218
E	Finite-Temperature Coordinate Frame	221
E.1	Application 1 (Sect. 3.5)	224
E.2	Application 2 (Sect. 3.6)	227

Contents XI

F Two-Loop Effective Action of QED at Zero Temperature	227
References	230
Index	237