

Contents

Introduction	1
References	3
Part I Mathematical Preliminaries	
1 Conformal Transformations and Conformal Killing Fields	7
1.1 Semi-Riemannian Manifolds	7
1.2 Conformal Transformations	9
1.3 Conformal Killing Fields	13
1.4 Classification of Conformal Transformations	15
1.4.1 Case 1: $n = p + q > 2$	15
1.4.2 Case 2: Euclidean Plane ($p = 2, q = 0$)	18
1.4.3 Case 3: Minkowski Plane ($p = q = 1$)	19
2 The Conformal Group	23
2.1 Conformal Compactification of $\mathbb{R}^{p,q}$	23
2.2 The Conformal Group of $\mathbb{R}^{p,q}$ for $p + q > 2$	28
2.3 The Conformal Group of $\mathbb{R}^{2,0}$	31
2.4 In What Sense Is the Conformal Group Infinite Dimensional?	33
2.5 The Conformal Group of $\mathbb{R}^{1,1}$	35
References	38
3 Central Extensions of Groups	39
3.1 Central Extensions	39
3.2 Quantization of Symmetries	44
3.3 Equivalence of Central Extensions	56
References	61
4 Central Extensions of Lie Algebras and Bargmann's Theorem	63
4.1 Central Extensions and Equivalence	63
4.2 Bargmann's Theorem	69
References	73

5	The Virasoro Algebra	75
5.1	Witt Algebra and Infinitesimal Conformal Transformations of the Minkowski Plane	75
5.2	Witt Algebra and Infinitesimal Conformal Transformations of the Euclidean Plane	77
5.3	The Virasoro Algebra as a Central Extension of the Witt Algebra . . .	79
5.4	Does There Exist a Complex Virasoro Group?	82
	References	84

Part II First Steps Toward Conformal Field Theory

6	Representation Theory of the Virasoro Algebra	91
6.1	Unitary and Highest-Weight Representations	91
6.2	Verma Modules	92
6.3	The Kac Determinant	95
6.4	Indecomposability and Irreducibility of Representations	99
6.5	Projective Representations of $\text{Diff}_+(\mathbb{S})$	100
	References	102
7	String Theory as a Conformal Field Theory	103
7.1	Classical Action Functionals and Equations of Motion for Strings . .	103
7.2	Canonical Quantization	111
7.3	Fock Space Representation of the Virasoro Algebra	115
7.4	Quantization of Strings	119
	References	120
8	Axioms of Relativistic Quantum Field Theory	121
8.1	Distributions	122
8.2	Field Operators	129
8.3	Wightman Axioms	131
8.4	Wightman Distributions and Reconstruction	137
8.5	Analytic Continuation and Wick Rotation	142
8.6	Euclidean Formulation	148
8.7	Conformal Covariance	149
	References	151
9	Foundations of Two-Dimensional Conformal Quantum Field Theory .	153
9.1	Axioms for Two-Dimensional Euclidean Quantum Field Theory . . .	153
9.2	Conformal Fields and the Energy–Momentum Tensor	159
9.3	Primary Fields, Operator Product Expansion, and Fusion	163
9.4	Other Approaches to Axiomatization	168
	References	169

- 10 Vertex Algebras** 171
 - 10.1 Formal Distributions 172
 - 10.2 Locality and Normal Ordering 177
 - 10.3 Fields and Locality 181
 - 10.4 The Concept of a Vertex Algebra 185
 - 10.5 Conformal Vertex Algebras 192
 - 10.6 Associativity of the Operator Product Expansion 199
 - 10.7 Induced Representations 209
 - References 212

- 11 Mathematical Aspects of the Verlinde Formula** 213
 - 11.1 The Moduli Space of Representations and Theta Functions 213
 - 11.2 The Verlinde Formula 219
 - 11.3 Fusion Rules for Surfaces with Marked Points 221
 - 11.4 Combinatorics on Fusion Rings: Verlinde Algebra 228
 - References 232

- A Some Further Developments** 235
 - References 236

- References** 239

- Index** 245

