
Contents

1	Convex Functions	1
1.1	Introduction	1
1.2	Convex Sets	1
1.2.1	Topological Properties of Convex Sets	3
1.2.2	Relative Interior of Convex Sets	4
1.2.3	Extreme Points and Extreme Directions	4
1.2.4	Supporting Hyperplanes and Separation Theorems.....	6
1.2.5	Convex Cones and Polarity	7
1.3	Convex Functions	10
1.3.1	Algebraic Structure of the Convex Functions	13
1.3.2	Composite Function	13
1.3.3	Differentiable and Twice Differentiable Convex Functions	14
1.4	Convexity and Homogeneity	16
1.5	Minima of Convex Functions	17
1.6	Exercises	18
1.7	References	21
2	Non-Differentiable Generalized Convex Functions	23
2.1	Introduction	23
2.2	Quasiconvexity and Strict Quasiconvexity	23
2.3	Semistrict Quasiconvexity	30
2.4	Generalized Convexity of Some Homogeneous Functions	34
2.4.1	The Cobb–Douglas Function	34
2.4.2	The Constant Elasticity of Substitution (C.E.S.) Function	35
2.4.3	The Leontief Production Function	35
2.4.4	A Generalized Cobb–Douglas Function	35
2.5	Generalized Quasiconvex Functions in One Variable.....	36
2.6	Exercises	38
2.7	References	40

3	Differentiable Generalized Convex Functions	41
3.1	Introduction	41
3.2	Differentiable Quasiconvex and Pseudoconvex Functions	41
3.2.1	Differentiable Quasiconvex Functions	41
3.2.2	Pseudoconvex Functions	43
3.2.3	Relationships	48
3.3	Quasilinearity and Pseudolinearity	50
3.3.1	Quasilinearity and Semistrict Quasilinearity	50
3.3.2	Pseudolinearity	53
3.4	Twice Differentiable Generalized Convex Functions	57
3.4.1	Quasiconvex Functions	57
3.4.2	Pseudoconvex Functions	60
3.4.3	Characterizations in Terms of the Bordered Hessian	61
3.5	Generalized Convexity at a Point	63
3.6	Exercises	68
3.7	References	71
4	Optimality and Generalized Convexity	73
4.1	Introduction	73
4.2	Necessary Optimality Conditions Via Separation Theorems	73
4.3	Generalized Convexity and Constraint Qualifications	78
4.4	Sufficiency of the Karush–Kuhn–Tucker Conditions	81
4.5	Local-Global Property	82
4.6	Maxima and Generalized Convexity	84
4.7	Minima, Maxima and Pseudolinearity	86
4.8	Economic Applications	87
4.8.1	The Utility Maximization Problem	89
4.8.2	The Expenditure Minimization Problem	91
4.8.3	The Profit Maximization Problem and the Cost Minimization Problem	91
4.9	Invex Functions	93
4.10	Exercises	95
4.11	References	97
5	Generalized Convexity and Generalized Monotonicity	99
5.1	Introduction	99
5.2	Concepts of Generalized Monotonicity	99
5.2.1	Differentiable Generalized Monotone Maps	103
5.3	Generalized Monotonicity of Maps of One Variable	103
5.4	Generalized Monotonicity of Affine Maps	105
5.5	Relationships Between Generalized Monotonicity and Generalized Convexity	108
5.6	The Generalized Charnes–Cooper Transformation	110
5.7	References	112

6	Generalized Convexity of Quadratic Functions	115
6.1	Introduction	115
6.2	Preliminary Results	115
6.2.1	Some Properties of a Quadratic form Associated with a Symmetric Matrix Having One Simple Negative Eigenvalue	116
6.3	Quadratic Functions	119
6.4	Quadratic Functions of Non-negative Variables	126
6.5	Pseudoconvexity on a Closed Set	128
6.5.1	Pseudoconvexity on the Non-negative Orthant	130
6.5.2	Generalized Convexity of a Quadratic form on \mathcal{R}_+^2	131
6.6	A Special Case	132
6.7	Exercises	134
6.8	References	135
7	Generalized Convexity of Some Classes of Fractional Functions	137
7.1	Introduction	137
7.2	The Ratio of a Quadratic and an Affine Function	137
7.3	The Sum of a Linear and a Linear Fractional Function	141
7.4	Pseudoconvexity and the Charnes–Cooper Variable Transformation	147
7.5	Sum of Two Linear Fractional Functions	149
7.6	Exercises	154
7.7	References	157
8	Sequential Methods for Generalized Convex Fractional Programs	159
8.1	Introduction	159
8.2	The Linear Fractional Problem	160
8.2.1	Isbell–Marlow’s Algorithm	162
8.2.2	Charnes–Cooper’s Algorithm	164
8.2.3	Martos’ Algorithm	166
8.2.4	Cambini–Martein’s Algorithm	168
8.2.5	The Case of an Unbounded Feasible Region	172
8.3	A Generalized Linear Fractional Problem	175
8.3.1	Sequential Methods	176
8.3.2	The Sum of Two Linear Fractional Functions	182
8.4	Generalized Linear Multiplicative Programs	183
8.4.1	The Sum of a Linear Function and the Product of Two Affine Functions	183
8.4.2	The Product Between an Affine Function and the Power of an Affine Function	185
8.5	The Optimal Level Solutions Method	189
8.6	References	193

9 Solutions	195
References	213
A Mathematical Review	229
A.1 Sets	229
A.2 The Euclidean Space \mathfrak{R}^n	230
A.3 Topological Concepts	234
A.4 Functions	235
B Concave and Generalized Concave Functions	241
Index	247