# Contents

**Foreword** .................................................... v

**List of Contributors** .................................................... xxv

**Introduction** .................................................... 1
Torben G. Andersen, Richard A. Davis, Jens-Peter Kreiss and Thomas Mikosch
References .................................................... 13

**Part I Recent Developments in GARCH Modeling**

**An Introduction to Univariate GARCH Models** .................................................... 17
Timo Teräsvirta
1 Introduction .................................................... 17
2 The ARCH Model .................................................... 18
3 The Generalized ARCH Model .................................................... 19
3.1 Why Generalized ARCH? .................................................... 19
3.2 Families of univariate GARCH models .................................................... 20
3.3 Nonlinear GARCH .................................................... 23
3.4 Time-varying GARCH .................................................... 26
3.5 Markov-switching ARCH and GARCH .................................................... 27
3.6 Integrated and fractionally integrated GARCH .................................................... 28
3.7 Semi- and nonparametric ARCH models .................................................... 30
3.8 GARCH-in-mean model .................................................... 30
3.9 Stylized facts and the first-order GARCH model .................................................... 31
4 Family of Exponential GARCH Models .................................................... 34
4.1 Definition and properties .................................................... 34
4.2 Stylized facts and the first-order EGARCH model .................................................... 35
4.3 Stochastic volatility .................................................... 36
5 Comparing EGARCH with GARCH .................................................... 37
6 Final Remarks and Further Reading .................................................... 38
References .................................................... 39

**Stationarity, Mixing, Distributional Properties and Moments of GARCH$(p, q)$–Processes** .................................................... 43
Alexander M. Lindner
1 Introduction .................................................... 43

2 Stationary Solutions . . . . . . . . . . 44
2.1 Strict stationarity of ARCH(1) and GARCH(1, 1) . 45
2.2 Strict stationarity of GARCH($p, q$) . . . . . . . 49
2.3 Ergodicity . . . . . . . . . . 52
2.4 Weak stationarity . . . . . . . . . . 53
3 The ARCH($\infty$) Representation and the Conditional Variance . . . . . . . . . . 54
4 Existence of Moments and the Autocovariance Function of the Squared Process . . . . . . . . . . 55
4.1 Moments of ARCH(1) and GARCH(1, 1) . . . . . . 56
4.2 Moments of GARCH($p, q$) . . . . . . . . . . 57
4.3 The autocorrelation function of the squares . . . . . . 60
5 Strong Mixing . . . . . . . . . . 62
6 Some Distributional Properties . . . . . . . . . . 64
7 Models Defined on the Non-Negative Integers . . . . . . 66
8 Conclusion . . . . . . . . . . 67
References . . . . . . . . . . 67

**ARCH($\infty$) Models and Long Memory Properties** . . . . . . 71
Liudas Giraitis, Remigijus Leipus and Donatas Surgailis
1 Introduction . . . . . . . . . . 71
2 Stationary ARCH($\infty$) Process . . . . . . . . . . 73
2.1 Volterra representations . . . . . . . . . . 73
2.2 Dependence structure, association, and central limit theorem . . . . . . . . . . 75
2.3 Infinite variance and integrated ARCH($\infty$) . . . . . . 77
3 Linear ARCH and Bilinear Model . . . . . . . . . . 79
References . . . . . . . . . . 82

**A Tour in the Asymptotic Theory of GARCH Estimation** . . . 85
Christian Francq and Jean-Michel Zakoïan
1 Introduction . . . . . . . . . . 85
2 Least–Squares Estimation of ARCH Models . . . . . . 87
3 Quasi–Maximum Likelihood Estimation . . . . . . 89
3.1 Pure GARCH models . . . . . . . . . . 90
3.2 ARMA–GARCH models . . . . . . . . . . 94
4 Efficient Estimation . . . . . . . . . . 95
5 Alternative Estimators . . . . . . . . . . 99
5.1 Self–weighted LSE for the ARMA parameters . . . . 100
5.2 Self–weighted QMLE . . . . . . . . . . 100
5.3 $L_p$–estimators . . . . . . . . . . 101
5.4 Least absolute deviations estimators . . . . . . 102
5.5 Whittle estimator . . . . . . . . . . 103
5.6 Moment estimators . . . . . . . . . . 104
6 Properties of Estimators when some GARCH Coefficients are Equal to Zero . . . . . . . . . . 104

6.1 Fitting an ARCH(1) model to a white noise ...... 105
6.2 On the need of additional assumptions ........... 106
6.3 Asymptotic distribution of the QMLE on the boundary ........................................ 106
6.4 Application to hypothesis testing ............... 107
7 Conclusion ................................................. 109
References ...................................................... 109

**Practical Issues in the Analysis of Univariate GARCH Models** 113
Eric Zivot
1 Introduction ............................................... 113
2 Some Stylized Facts of Asset Returns ................... 114
3 The ARCH and GARCH Model ........................... 115
3.1 Conditional mean specification .................. 118
3.2 Explanatory variables in the conditional variance equation ...................................... 119
3.3 The GARCH model and stylized facts of asset returns ....................................... 119
3.4 Temporal aggregation ......................... 121
4 Testing for ARCH/GARCH Effects ..................... 121
4.1 Testing for ARCH effects in daily and monthly returns ....................................... 122
5 Estimation of GARCH Models ......................... 123
5.1 Numerical accuracy of GARCH estimates ........ 125
5.2 Quasi-maximum likelihood estimation ........... 126
5.3 Model selection .............................. 126
5.4 Evaluation of estimated GARCH models ......... 127
5.5 Estimation of GARCH models for daily and monthly returns ............................. 127
6 GARCH Model Extensions ............................ 131
6.1 Asymmetric leverage effects and news impact ..... 131
6.2 Non-Gaussian error distributions ................ 135
7 Long Memory GARCH Models......................... 137
7.1 Testing for long memory ....................... 139
7.2 Two component GARCH model................. 139
7.3 Integrated GARCH model ..................... 140
7.4 Long memory GARCH models for daily returns ... 141
8 GARCH Model Prediction............................. 142
8.1 GARCH and forecasts for the conditional mean ... 142
8.2 Forecasts from the GARCH(1,1) model .......... 143
8.3 Forecasts from asymmetric GARCH(1,1) models .. 144
8.4 Simulation-based forecasts ..................... 145
8.5 Forecasting the volatility of multiperiod returns ... 145
8.6 Evaluating volatility predictions ............... 146

8.7 Forecasting the volatility of Microsoft and the S&P 500 . . . . . 150
9 Final Remarks . . . . . 151
References . . . . . 151

**Semiparametric and Nonparametric ARCH Modeling** . . . . . 157
Oliver B. Linton
1 Introduction . . . . . 157
2 The GARCH Model . . . . . 157
3 The Nonparametric Approach . . . . . 158
3.1 Error density . . . . . 158
3.2 Functional form of volatility function . . . . . 159
3.3 Relationship between mean and variance . . . . . 162
3.4 Long memory . . . . . 163
3.5 Locally stationary processes . . . . . 164
3.6 Continuous time . . . . . 164
4 Conclusion . . . . . 165
References . . . . . 165

**Varying Coefficient GARCH Models** . . . . . 169
Pavel Čížek and Vladimir Spokoiny
1 Introduction . . . . . 169
2 Conditional Heteroscedasticity Models . . . . . 171
2.1 Model estimation . . . . . 173
2.2 Test of homogeneity against a change–point alternative . . . . . 173
3 Adaptive Nonparametric Estimation . . . . . 175
3.1 Adaptive choice of the interval of homogeneity . . . . . 176
3.2 Parameters of the method and the implementation details . . . . . 176
4 Real–Data Application . . . . . 179
4.1 Finite–sample critical values for the test of homogeneity . . . . . 179
4.2 Stock index S&P 500 . . . . . 180
5 Conclusion . . . . . 183
References . . . . . 183

**Extreme Value Theory for GARCH Processes** . . . . . 187
Richard A. Davis and Thomas Mikosch
1 The Model . . . . . 187
2 Strict Stationarity and Mixing Properties . . . . . 188
3 Embedding a GARCH Process in a Stochastic Recurrence Equation . . . . . 189
4 The Tails of a GARCH Process . . . . . 190
5 Limit Theory for Extremes . . . . . 194
5.1 Convergence of maxima . . . . . 194

5.2 Convergence of point processes ................ 195
5.3 The behavior of the sample autocovariance function 197
References ........................................ 199

**Multivariate GARCH Models** ........................ 201
Annastiina Silvennoinen and Timo Teräsvirta
1 Introduction ...................................... 201
2 Models ............................................ 203
2.1 Models of the conditional covariance matrix ...... 204
2.2 Factor models .................................... 207
2.3 Models of conditional variances and correlations .. 210
2.4 Nonparametric and semiparametric approaches ... 215
3 Statistical Properties ............................. 218
4 Hypothesis Testing in Multivariate GARCH Models ...... 218
4.1 General misspecification tests .................. 219
4.2 Tests for extensions of the CCC–GARCH model .. 221
5 An Application .................................... 222
6 Final Remarks ..................................... 224
References ........................................ 226

**Part II Recent Developments in Stochastic Volatility Modeling**

**Stochastic Volatility: Origins and Overview** ................. 233
Neil Shephard and Torben G. Andersen
1 Introduction ...................................... 233
2 The Origin of SV Models ........................... 235
3 Second Generation Model Building ................... 240
3.1 Univariate models ................................ 240
3.2 Multivariate models .............................. 241
4 Inference Based on Return Data ..................... 242
4.1 Moment–based inference .......................... 242
4.2 Simulation–based inference ....................... 243
5 Options ........................................... 246
5.1 Models ........................................... 246
6 Realized Volatility ................................ 247
References ........................................ 250

**Probabilistic Properties of Stochastic Volatility Models** ...... 255
Richard A. Davis and Thomas Mikosch
1 The Model ......................................... 255
2 Stationarity, Ergodicity and Strong Mixing ............. 256
2.1 Strict stationarity ............................... 256
2.2 Ergodicity and strong mixing ..................... 257
3 The Covariance Structure ........................... 258
4 Moments and Tails .................................. 261
5 Asymptotic Theory for the Sample ACVF and ACF ...... 263

References .......... 266

**Moment–Based Estimation of Stochastic Volatility Models** ... 269
Eric Renault
1 Introduction .......... 270
2 The Use of a Regression Model to Analyze Fluctuations in Variance .......... 272
2.1 The linear regression model for conditional variance 272
2.2 The SR–SARV(p) model .......... 274
2.3 The Exponential SARV model .......... 277
2.4 Other parametric SARV models .......... 279
3 Implications of SV Model Specification for Higher Order Moments .......... 281
3.1 Fat tails and variance of the variance .......... 281
3.2 Skewness, feedback and leverage effects .......... 284
4 Continuous Time Models .......... 286
4.1 Measuring volatility .......... 287
4.2 Moment-based estimation with realized volatility .. 288
4.3 Reduced form models of volatility .......... 292
4.4 High frequency data with random times separating successive observations .......... 293
5 Simulation–Based Estimation .......... 295
5.1 Simulation-based bias correction .......... 296
5.2 Simulation-based indirect inference .......... 298
5.3 Simulated method of moments .......... 300
5.4 Indirect inference in presence of misspecification .. 304
6 Concluding Remarks .......... 305
References .......... 307

**Parameter Estimation and Practical Aspects of Modeling Stochastic Volatility** .......... 313
Borus Jungbacker and Siem Jan Koopman
1 Introduction .......... 313
2 A Quasi-Likelihood Analysis Based on Kalman Filter Methods .......... 316
2.1 Kalman filter for prediction and likelihood evaluation .......... 319
2.2 Smoothing methods for the conditional mean, variance and mode .......... 320
2.3 Practical considerations for analyzing the linearized SV model .......... 321
3 A Monte Carlo Likelihood Analysis .......... 322
3.1 Construction of a proposal density .......... 323
3.2 Sampling from the importance density and Monte Carlo likelihood .......... 325
4 Some Generalizations of SV Models .......... 327

4.1 Basic SV model . . . . . 327
4.2 Multiple volatility factors . . . . . 328
4.3 Regression and fixed effects . . . . . 329
4.4 Heavy-tailed innovations . . . . . 330
4.5 Additive noise . . . . . 331
4.6 Leverage effects . . . . . 331
4.7 Stochastic volatility in mean . . . . . 333
5 Empirical Illustrations . . . . . 333
5.1 Standard & Poor's 500 stock index: volatility estimation . . . . . 334
5.2 Standard & Poor's 500 stock index: regression effects . . . . . 335
5.3 Daily changes in exchange rates: dollar–pound and dollar–yen . . . . . 337
6 Conclusions . . . . . 340
Appendix . . . . . 340
References . . . . . 342

**Stochastic Volatility Models with Long Memory** . . . . . 345
Clifford M. Hurvich and Philippe Soulier
1 Introduction . . . . . 345
2 Basic Properties of the LMSV Model . . . . . 346
3 Parametric Estimation . . . . . 347
4 Semiparametric Estimation . . . . . 349
5 Generalizations of the LMSV Model . . . . . 352
6 Applications of the LMSV Model . . . . . 352
References . . . . . 353

**Extremes of Stochastic Volatility Models** . . . . . 355
Richard A. Davis and Thomas Mikosch
1 Introduction . . . . . 355
2 The Tail Behavior of the Marginal Distribution . . . . . 356
2.1 The light-tailed case . . . . . 356
2.2 The heavy-tailed case . . . . . 357
3 Point Process Convergence . . . . . 358
3.1 Background . . . . . 358
3.2 Application to stochastic volatility models . . . . . 360
References . . . . . 364

**Multivariate Stochastic Volatility** . . . . . 365
Siddhartha Chib, Yasuhiro Omori and Manabu Asai
1 Introduction . . . . . 366
2 Basic MSV Model . . . . . 369
2.1 No-leverage model . . . . . 369
2.2 Leverage effects . . . . . 373
2.3 Heavy-tailed measurement error models . . . . . 377

3 Factor MSV Model ... 379
3.1 Volatility factor model ... 379
3.2 Mean factor model ... 382
3.3 Bayesian analysis of mean factor MSV model ... 384
4 Dynamic Correlation MSV Model ... 388
4.1 Modeling by reparameterization ... 388
4.2 Matrix exponential transformation ... 390
4.3 Wishart process ... 391
5 Conclusion ... 396
References ... 397

**Part III Topics in Continuous Time Processes**

**An Overview of Asset–Price Models** ... 403
Peter J. Brockwell
1 Introduction ... 404
2 Shortcomings of the BSM Model ... 409
3 A General Framework for Option Pricing ... 410
4 Some Non-Gaussian Models for Asset Prices ... 411
5 Further Models ... 415
References ... 416

**Ornstein–Uhlenbeck Processes and Extensions** ... 421
Ross A. Maller, Gernot Müller and Alex Szimayer
1 Introduction ... 422
2 OU Process Driven by Brownian Motion ... 422
3 Generalised OU Processes ... 424
3.1 Background on bivariate Lévy processes ... 424
3.2 Lévy OU processes ... 426
3.3 Self-decomposability, self-similarity, class $L$, Lamperti transform ... 429
4 Discretisations ... 430
4.1 Autoregressive representation, and perpetuities ... 430
4.2 Statistical issues: Estimation and hypothesis testing 431
4.3 Discretely sampled process ... 431
4.4 Approximating the COGARCH ... 432
5 Conclusion ... 435
References ... 435

**Jump–Type Lévy Processes** ... 439
Ernst Eberlein
1 Probabilistic Structure of Lévy Processes ... 439
2 Distributional Description of Lévy Processes ... 443
3 Financial Modeling ... 446
4 Examples of Lévy Processes with Jumps ... 449
4.1 Poisson and compound Poisson processes ... 449

4.2 Lévy jump diffusion . . . . . . . . . . 450
4.3 Hyperbolic Lévy processes . . . . . . . . . . 450
4.4 Generalized hyperbolic Lévy processes . . . . . . . . . . 451
4.5 CGMY and variance gamma Lévy processes . . . . . . 452
4.6 $\alpha$-Stable Lévy processes . . . . . . . . . . 453
4.7 Meixner Lévy processes . . . . . . . . . . 453
References . . . . . . . . . . 454

**Lévy–Driven Continuous–Time ARMA Processes** . . . . . . . . . . 457
Peter J. Brockwell
1 Introduction . . . . . . . . . . 458
2 Second–Order Lévy–Driven CARMA Processes . . . . . . . . . . 460
3 Connections with Discrete–Time ARMA Processes . . . . . . . 470
4 An Application to Stochastic Volatility Modelling . . . . . . . . 474
5 Continuous–Time GARCH Processes . . . . . . . . . . 476
6 Inference for CARMA Processes . . . . . . . . . . 478
References . . . . . . . . . . 479

**Continuous Time Approximations to GARCH and Stochastic Volatility Models** . . . . . . . . . . 481
Alexander M. Lindner
1 Stochastic Volatility Models and Discrete GARCH . . . . . . . 481
2 Continuous Time GARCH Approximations . . . . . . . . . . 482
2.1 Preserving the random recurrence equation property 483
2.2 The diffusion limit of Nelson . . . . . . . . . . 484
2.3 The COGARCH model . . . . . . . . . . 486
2.4 Weak GARCH processes . . . . . . . . . . 488
2.5 Stochastic delay equations . . . . . . . . . . 489
2.6 A continuous time GARCH model designed for option pricing . . . . . . . . . . 490
3 Continuous Time Stochastic Volatility Approximations . . . . 491
3.1 Sampling a continuous time SV model at equidistant times . . . . . . . . . . 491
3.2 Approximating a continuous time SV model . . . . . . 493
References . . . . . . . . . . 495

**Maximum Likelihood and Gaussian Estimation of Continuous Time Models in Finance** . . . . . . . . . . 497
Peter C. B. Phillips and Jun Yu
1 Introduction . . . . . . . . . . 498
2 Exact ML Methods . . . . . . . . . . 499
2.1 ML based on the transition density . . . . . . . . . . 499
2.2 ML based on the continuous record likelihood . . . . 502
3 Approximate ML Methods Based on Transition Densities . . 503
3.1 The Euler approximation and refinements . . . . . . . . 504
3.2 Closed–form approximations . . . . . . . . . . 509

3.3 Simulated infill ML methods ... 512
3.4 Other approaches ... 514
4 Approximate ML Methods Based on the Continuous Record Likelihood and Realized Volatility ... 516
5 Monte Carlo Simulations ... 519
6 Estimation Bias Reduction Techniques ... 520
6.1 Jackknife estimation ... 521
6.2 Indirect inference estimation ... 522
7 Multivariate Continuous Time Models ... 524
8 Conclusions ... 527
References ... 527

**Parametric Inference for Discretely Sampled Stochastic Differential Equations** ... 531
Michael Sørensen
1 Introduction ... 531
2 Asymptotics: Fixed Frequency ... 532
3 Likelihood Inference ... 536
4 Martingale Estimating Functions ... 538
5 Explicit Inference ... 543
6 High Frequency Asymptotics and Efficient Estimation ... 548
References ... 551

**Realized Volatility** ... 555
Torben G. Andersen and Luca Benzoni
1 Introduction ... 556
2 Measuring Mean Return versus Return Volatility ... 557
3 Quadratic Return Variation and Realized Volatility ... 559
4 Conditional Return Variance and Realized Volatility ... 561
5 Jumps and Bipower Variation ... 563
6 Efficient Sampling versus Microstructure Noise ... 564
7 Empirical Applications ... 566
7.1 Early work ... 566
7.2 Volatility forecasting ... 567
7.3 The distributional implications of the no-arbitrage condition ... 568
7.4 Multivariate quadratic variation measures ... 568
7.5 Realized volatility, model specification and estimation ... 569
8 Possible Directions for Future Research ... 569
References ... 570

**Estimating Volatility in the Presence of Market Microstructure Noise: A Review of the Theory and Practical Considerations** ........ 577
Yacine Aït-Sahalia and Per A. Mykland
1 Introduction ........ 577
2 Estimators ........ 579
2.1 The parametric volatility case ........ 579
2.2 The nonparametric stochastic volatility case ........ 582
3 Refinements ........ 585
3.1 Multi-scale realized volatility ........ 585
3.2 Non-equally spaced observations ........ 586
3.3 Serially-correlated noise ........ 587
3.4 Noise correlated with the price signal ........ 589
3.5 Small sample edgeworth expansions ........ 591
3.6 Robustness to departures from the data generating process assumptions ........ 591
4 Computational and Practical Implementation Considerations ........ 592
4.1 Calendar, tick and transaction time sampling ........ 592
4.2 Transactions or quotes ........ 592
4.3 Selecting the number of subsamples in practice ........ 593
4.4 High versus low liquidity assets ........ 594
4.5 Robustness to data cleaning procedures ........ 594
4.6 Smoothing by averaging ........ 595
5 Conclusions ........ 596
References ........ 596

**Option Pricing** ........ 599
Jan Kallsen
1 Introduction ........ 599
2 Arbitrage Theory from a Market Perspective ........ 600
3 Martingale Modelling ........ 603
4 Arbitrage Theory from an Individual Perspective ........ 605
5 Quadratic Hedging ........ 606
6 Utility Indifference Pricing ........ 607
References ........ 611

**An Overview of Interest Rate Theory** ........ 615
Tomas Björk
1 General Background ........ 615
2 Interest Rates and the Bond Market ........ 618
3 Factor Models ........ 620
4 Modeling under the Objective Measure $P$ ........ 621
4.1 The market price of risk ........ 622
5 Martingale Modeling ........ 623
5.1 Affine term structures ........ 624

5.2 Short rate models ........ 625
5.3 Inverting the yield curve ........ 627
6 Forward Rate Models ........ 629
6.1 The HJM drift condition ........ 629
6.2 The Musiela parameterization ........ 631
7 Change of Numeraire ........ 632
7.1 Generalities ........ 632
7.2 Forward measures ........ 635
7.3 Option pricing ........ 635
8 LIBOR Market Models ........ 638
8.1 Caps: definition and market practice ........ 638
8.2 The LIBOR market model ........ 640
8.3 Pricing caps in the LIBOR model ........ 641
8.4 Terminal measure dynamics and existence ........ 641
9 Potentials and Positive Interest ........ 642
9.1 Generalities ........ 642
9.2 The Flesaker–Hughston fractional model ........ 644
9.3 Connections to the Riesz decomposition ........ 646
9.4 Conditional variance potentials ........ 647
9.5 The Rogers Markov potential approach ........ 648
10 Notes ........ 650
References ........ 651

**Extremes of Continuous–Time Processes** ........ 653
Vicky Fasen
1 Introduction ........ 653
2 Extreme Value Theory ........ 654
2.1 Extremes of discrete–time processes ........ 655
2.2 Extremes of continuous–time processes ........ 656
2.3 Extensions ........ 656
3 The Generalized Ornstein-Uhlenbeck (GOU)–Model ........ 657
3.1 The Ornstein–Uhlenbeck process ........ 658
3.2 The non–Ornstein–Uhlenbeck process ........ 659
3.3 Comparison of the models ........ 661
4 Tail Behavior of the Sample Maximum ........ 661
5 Running sample Maxima and Extremal Index Function ........ 663
6 Conclusion ........ 664
References ........ 665

**Part IV Topics in Cointegration and Unit Roots**

**Cointegration: Overview and Development** ........ 671
Søren Johansen
1 Introduction ........ 671
1.1 Two examples of cointegration ........ 672

1.2 Three ways of modeling cointegration ............ 673
1.3 The model analyzed in this article .............. 674
2 Integration, Cointegration and Granger's Representation Theorem ............................................ 675
2.1 Definition of integration and cointegration ....... 675
2.2 The Granger Representation Theorem ........... 677
2.3 Interpretation of cointegrating coefficients ........ 678
3 Interpretation of the $I(1)$ Model for Cointegration ........ 680
3.1 The models $H(r)$ ............................ 680
3.2 Normalization of parameters of the $I(1)$ model.... 681
3.3 Hypotheses on long-run coefficients.............. 681
3.4 Hypotheses on adjustment coefficients ........... 682
4 Likelihood Analysis of the $I(1)$ Model.................. 683
4.1 Checking the specifications of the model ......... 683
4.2 Reduced rank regression ....................... 683
4.3 Maximum likelihood estimation in the $I(1)$ model and derivation of the rank test.................. 684
5 Asymptotic Analysis.................................. 686
5.1 Asymptotic distribution of the rank test ......... 686
5.2 Asymptotic distribution of the estimators ........ 687
6 Further Topics in the Area of Cointegration ............ 689
6.1 Rational expectations ......................... 689
6.2 The $I(2)$ model ............................... 690
7 Concluding Remarks ................................. 691
References ........................................... 692

**Time Series with Roots on or Near the Unit Circle** .......... 695
Ngai Hang Chan
1 Introduction......................................... 695
2 Unit Root Models.................................... 696
2.1 First order .................................... 697
2.2 AR($p$) models ................................ 699
2.3 Model selection ................................ 702
3 Miscellaneous Developments and Conclusion ............. 704
References ........................................... 705

**Fractional Cointegration** ................................ 709
Willa W. Chen and Clifford M. Hurvich
1 Introduction......................................... 709
2 Type I and Type II Definitions of $I(d)$ .................. 710
2.1 Univariate series ............................... 710
2.2 Multivariate series ............................. 713
3 Models for Fractional Cointegration ................... 715
3.1 Parametric models ............................. 716
4 Tapering ............................................ 717
5 Semiparametric Estimation of the Cointegrating Vectors .. 718

6 Testing for Cointegration; Determination of Cointegrating Rank ..... 723
References ..... 724

## Part V Special Topics – Risk

**Different Kinds of Risk** ..... 729
Paul Embrechts, Hansjörg Furrer and Roger Kaufmann
1 Introduction ..... 729
2 Preliminaries ..... 732
2.1 Risk measures ..... 732
2.2 Risk factor mapping and loss portfolios ..... 735
3 Credit Risk ..... 736
3.1 Structural models ..... 737
3.2 Reduced form models ..... 737
3.3 Credit risk for regulatory reporting ..... 738
4 Market Risk ..... 738
4.1 Market risk models ..... 739
4.2 Conditional versus unconditional modeling ..... 740
4.3 Scaling of market risks ..... 740
5 Operational Risk ..... 742
6 Insurance Risk ..... 744
6.1 Life insurance risk ..... 744
6.2 Modeling parametric life insurance risk ..... 745
6.3 Non-life insurance risk ..... 747
7 Aggregation of Risks ..... 748
8 Summary ..... 749
References ..... 750

**Value–at–Risk Models** ..... 753
Peter Christoffersen
1 Introduction and Stylized Facts ..... 753
2 A Univariate Portfolio Risk Model ..... 755
2.1 The dynamic conditional variance model ..... 756
2.2 Univariate filtered historical simulation ..... 757
2.3 Univariate extensions and alternatives ..... 759
3 Multivariate, Base–Asset Return Methods ..... 760
3.1 The dynamic conditional correlation model ..... 761
3.2 Multivariate filtered historical simulation ..... 761
3.3 Multivariate extensions and alternatives ..... 763
4 Summary and Further Issues ..... 764
References ..... 764

**Copula–Based Models for Financial Time Series** . . . . . . . . . . . . . 767
Andrew J. Patton
1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 767
2 Copula–Based Models for Time Series . . . . . . . . . . . . . . . . . . 771
2.1 Copula–based models for multivariate time series . 772
2.2 Copula–based models for univariate time series . . . 773
2.3 Estimation and evaluation of copula–based models for time series . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 775
3 Applications of Copulas in Finance and Economics . . . . . . . 778
4 Conclusions and Areas for Future Research . . . . . . . . . . . . . . 780
References . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 781

**Credit Risk Modeling** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 787
David Lando
1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 787
2 Modeling the Probability of Default and Recovery . . . . . . . . 788
3 Two Modeling Frameworks . . . . . . . . . . . . . . . . . . . . . . . . . . . 789
4 Credit Default Swap Spreads . . . . . . . . . . . . . . . . . . . . . . . . . 792
5 Corporate Bond Spreads and Bond Returns . . . . . . . . . . . . . 795
6 Credit Risk Correlation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 795
References . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 797

**Part V Special Topics – Time Series Methods**

**Evaluating Volatility and Correlation Forecasts** . . . . . . . . . . . . . . 801
Andrew J. Patton and Kevin Sheppard
1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 801
1.1 Notation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 803
2 Direct Evaluation of Volatility Forecasts . . . . . . . . . . . . . . . . 804
2.1 Forecast optimality tests for univariate volatility forecasts . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 805
2.2 MZ regressions on transformations of $\widehat{\sigma}_t^2$ . . . . . . . . . 806
2.3 Forecast optimality tests for multivariate volatility forecasts . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 807
2.4 Improved MZ regressions using generalised least squares . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 808
2.5 Simulation study . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 810
3 Direct Comparison of Volatility Forecasts . . . . . . . . . . . . . . . 815
3.1 Pair–wise comparison of volatility forecasts . . . . . . . 816
3.2 Comparison of many volatility forecasts . . . . . . . . . . 817
3.3 ‘Robust’ loss functions for forecast comparison . . . . 818
3.4 Problems arising from ‘non–robust’ loss functions . 819
3.5 Choosing a “robust” loss function . . . . . . . . . . . . . . . 823
3.6 Robust loss functions for multivariate volatility comparison . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 825

3.7 Direct comparison via encompassing tests ........ 828
4 Indirect Evaluation of Volatility Forecasts ............... 830
4.1 Portfolio optimisation ........................ 831
4.2 Tracking error minimisation .................... 832
4.3 Other methods of indirect evaluation ............ 833
5 Conclusion ..................................... 835
References ........................................... 835

**Structural Breaks in Financial Time Series** ................. 839
Elena Andreou and Eric Ghysels
1 Introduction ......................................... 839
2 Consequences of Structural Breaks in Financial Time Series 840
3 Methods for Detecting Structural Breaks ................ 843
3.1 Assumptions ................................. 844
3.2 Historical and sequential partial–sums change–point statistics ......................... 845
3.3 Multiple breaks tests ......................... 848
4 Change–Point Tests in Returns and Volatility ............ 851
4.1 Tests based on empirical volatility processes ...... 851
4.2 Empirical processes and the SV class of models ... 854
4.3 Tests based on parametric volatility models ...... 858
4.4 Change–point tests in long memory ............ 861
4.5 Change–point in the distribution ................ 863
5 Conclusions ......................................... 865
References ........................................... 866

**An Introduction to Regime Switching Time Series Models** ... 871
Theis Lange and Anders Rahbek
1 Introduction ......................................... 871
1.1 Markov and observation switching .............. 872
2 Switching ARCH and CVAR ........................... 874
2.1 Switching ARCH and GARCH ................ 875
2.2 Switching CVAR ............................. 877
3 Likelihood–Based Estimation ......................... 879
4 Hypothesis Testing .................................. 881
5 Conclusion ........................................... 883
References ........................................... 883

**Model Selection** ............................................ 889
Hannes Leeb and Benedikt M. Pötscher
1 The Model Selection Problem ......................... 889
1.1 A general formulation ........................ 889
1.2 Model selection procedures .................... 892
2 Properties of Model Selection Procedures and of Post–Model–Selection Estimators ....................... 900
2.1 Selection probabilities and consistency ........... 900

2.2 Risk properties of post-model-selection estimators 903
2.3 Distributional properties of post-model-selection estimators 906
3 Model Selection in Large- or Infinite–Dimensional Models 908
4 Related Procedures Based on Shrinkage and Model Averaging 915
5 Further Reading 916
References 916

**Nonparametric Modeling in Financial Time Series** 927
Jürgen Franke, Jens-Peter Kreiss and Enno Mammen
1 Introduction 927
2 Nonparametric Smoothing for Time Series 929
2.1 Density estimation via kernel smoothing 929
2.2 Kernel smoothing regression 932
2.3 Diffusions 935
3 Testing 937
4 Nonparametric Quantile Estimation 940
5 Advanced Nonparametric Modeling 942
6 Sieve Methods 944
References 947

**Modelling Financial High Frequency Data Using Point Processes** 953
Luc Bauwens and Nikolaus Hautsch
1 Introduction 953
2 Fundamental Concepts of Point Process Theory 954
2.1 Notation and definitions 955
2.2 Compensators, intensities, and hazard rates 955
2.3 Types and representations of point processes 956
2.4 The random time change theorem 959
3 Dynamic Duration Models 960
3.1 ACD models 960
3.2 Statistical inference 963
3.3 Other models 964
3.4 Applications 965
4 Dynamic Intensity Models 967
4.1 Hawkes processes 967
4.2 Autoregressive intensity processes 969
4.3 Statistical inference 973
4.4 Applications 975
References 976

## Part V Special Topics – Simulation Based Methods

**Resampling and Subsampling for Financial Time Series** ...... 983
Efstathios Paparoditis and Dimitris N. Politis
1 Introduction ... 983
2 Resampling the Time Series of Log–Returns ... 986
2.1 Parametric methods based on i.i.d. resampling of residuals ... 986
2.2 Nonparametric methods based on i.i.d. resampling of residuals ... 988
2.3 Markovian bootstrap ... 990
3 Resampling Statistics Based on the Time Series of Log–Returns ... 992
3.1 Regression bootstrap ... 992
3.2 Wild bootstrap ... 993
3.3 Local bootstrap ... 994
4 Subsampling and Self–Normalization ... 995
References ... 997

**Markov Chain Monte Carlo** ... 1001
Michael Johannes and Nicholas Polson
1 Introduction ... 1001
2 Overview of MCMC Methods ... 1002
2.1 Clifford–Hammersley theorem ... 1002
2.2 Constructing Markov chains ... 1003
2.3 Convergence theory ... 1007
3 Financial Time Series Examples ... 1008
3.1 Geometric Brownian motion ... 1008
3.2 Time-varying expected returns ... 1009
3.3 Stochastic volatility models ... 1010
4 Further Reading ... 1011
References ... 1012

**Particle Filtering** ... 1015
Michael Johannes and Nicholas Polson
1 Introduction ... 1015
2 A Motivating Example ... 1017
3 Particle Filters ... 1019
3.1 Exact particle filtering ... 1021
3.2 SIR ... 1024
3.3 Auxiliary particle filtering algorithms ... 1026
4 Further Reading ... 1027
References ... 1028

**Index** ... 1031