

---

# Contents

General Notations .....	XIX
-------------------------	-----

---

## Part I General

---

<b>1 Introduction</b> .....	3
1.1 Brief Historical Survey .....	3
1.1.1 Particle Penetration .....	3
1.1.2 Radiation Effects .....	9
1.2 Applications .....	10
1.2.1 Fundamental Physics Research .....	11
1.2.2 Astrophysics and Space Science .....	11
1.2.3 Plasma Physics and Fusion Research .....	12
1.2.4 Materials Research and Engineering .....	12
1.2.5 Analytical Chemistry .....	16
1.2.6 Biomedical Research .....	17
1.3 Measurements and Experimental Tools .....	18
1.3.1 Sources of Energetic Charged Particles .....	18
1.3.2 Targets and Detecting Devices .....	19
1.4 General-Physics and Related Aspects .....	20
1.5 Literature .....	21
1.6 Nomenclature .....	22
Problems .....	22
References .....	24
<b>2 Elementary Penetration Theory</b> .....	27
2.1 Introductory Comments .....	27
2.2 Collision Statistics .....	28
2.2.1 Definition of Cross Section .....	28
2.2.2 Multiple Collisions; Poisson's Formula .....	31
2.2.3 Energy Loss .....	33

2.2.4	Energy-Loss Straggling . . . . .	34
2.2.5	Differential Cross Section . . . . .	36
2.2.6	Range . . . . .	36
2.3	Electronic and Nuclear Stopping . . . . .	38
2.3.1	General Considerations . . . . .	38
2.3.2	Free-Coulomb Collision . . . . .	39
2.3.3	Stopping and Straggling . . . . .	42
2.3.4	Adiabatic Limit to Electronic Stopping . . . . .	44
2.3.5	Relativistic Extension . . . . .	46
2.3.6	Validity of Classical-Orbit Picture . . . . .	47
2.3.7	Screening in Nuclear Stopping . . . . .	49
2.4	Multiple Scattering . . . . .	51
2.4.1	Small-Angle Approximation . . . . .	51
2.4.2	Statistics . . . . .	52
2.4.3	Nuclear and Electronic Scattering . . . . .	54
2.5	Estimates . . . . .	56
2.5.1	Alpha Particles . . . . .	56
2.5.2	Preview: Energy and $Z_1$ dependence . . . . .	57
2.6	Electron and Positron Penetration . . . . .	58
2.7	Discussion and Outlook . . . . .	60
	Problems . . . . .	61
	References . . . . .	64
<b>3</b>	<b>Elastic Scattering . . . . .</b>	<b>67</b>
3.1	Introductory Comments . . . . .	67
3.2	Conservation Laws . . . . .	68
3.2.1	Nonrelativistic Regime . . . . .	68
3.2.2	Relativistic Regime . . . . .	71
3.3	Classical Scattering Theory . . . . .	74
3.3.1	The Scattering Integral . . . . .	74
3.3.2	Runge-Lenz Vector and Rutherford's Law . . . . .	76
3.3.3	Scaling Relations . . . . .	79
3.3.4	Time Integral ( $\star$ ) . . . . .	80
3.3.5	Relativistic Scattering Integral ( $\star$ ) . . . . .	82
3.3.6	Perturbation Theory ( $\star$ ) . . . . .	84
3.4	Quantum Theory of Elastic Scattering . . . . .	85
3.4.1	Laboratory and Center-of-Mass Variables . . . . .	85
3.4.2	Scattering Amplitude and Differential Cross Section . . . . .	86
3.4.3	Born Approximation . . . . .	88
3.4.4	Partial-Wave Expansion . . . . .	92
3.5	Coulomb Scattering . . . . .	97
3.5.1	Phase Shifts . . . . .	97
3.5.2	Cross Section ( $\star$ ) . . . . .	98
3.5.3	Relativistic Extension . . . . .	100
3.6	Discussion and Outlook . . . . .	101

Problems ..... 101  
 References ..... 104

**Part II Stopping**

**4 Stopping of Swift Point Charge I: Bohr and Bethe Theory** . 109  
 4.1 Introductory Comments ..... 109  
 4.2 Classical Perturbation Theory ..... 110  
     4.2.1 Energy Transfer to Harmonic Oscillator ..... 110  
     4.2.2 Distant Collisions: Dipole Approximation ..... 112  
     4.2.3 Relativistic Extension ..... 114  
 4.3 Semiclassical Theory ..... 115  
     4.3.1 General Considerations ..... 115  
     4.3.2 Time-Dependent Perturbation Theory ..... 116  
     4.3.3 Distant Collisions ..... 117  
     4.3.4 Excitation Cross Section ..... 119  
 4.4 Plane-Wave Born Approximation ..... 120  
     4.4.1 General Considerations ..... 120  
     4.4.2 Stationary Perturbation Theory ..... 120  
     4.4.3 Excitation Cross Section ..... 122  
     4.4.4 Coulomb Interaction ..... 124  
 4.5 The Stopping Cross Section ..... 124  
     4.5.1 Bohr Stopping Formula ..... 124  
     4.5.2 Semiclassical Theory: Harmonic Oscillator ..... 129  
     4.5.3 Plane-Wave Born Approximation ..... 131  
     4.5.4 Bethe Stopping Formula ..... 132  
     4.5.5 Mean Logarithmic Excitation Energy ..... 135  
 4.6 Discussion and Outlook ..... 135  
 Problems ..... 137  
 References ..... 138

**5 Dielectric Stopping Theory** ..... 141  
 5.1 Introductory Comments ..... 141  
 5.2 Electrodynamics ..... 142  
     5.2.1 Field Equations in Vacuum ..... 142  
     5.2.2 Linear Response ..... 144  
     5.2.3 Connection to Stopping Force ..... 145  
 5.3 Gaseous Medium ..... 146  
     5.3.1 Dielectric Function ..... 147  
     5.3.2 Bethe Stopping Formula ..... 148  
     5.3.3 Nonrelativistic Density Effect ..... 149  
 5.4 Static Electron Gas ..... 151  
     5.4.1 Dielectric Function ..... 151  
     5.4.2 Relativistic Extension (★) ..... 152

5.4.3	Stopping Force . . . . .	153
5.4.4	Oscillator Strength, Equipartition Rule and Differential Cross Section . . . . .	155
5.4.5	Plasmon-Pole Approximation (★) . . . . .	157
5.5	Assembly of Harmonic Oscillators (★) . . . . .	157
5.5.1	Dielectric Function . . . . .	157
5.5.2	Excitation Spectrum . . . . .	158
5.5.3	Stopping Force . . . . .	159
5.6	Relativistic Bethe Stopping Theory (★) . . . . .	159
5.6.1	Regimes of Momentum Transfer . . . . .	159
5.6.2	Transverse Field: Low Momentum Transfers . . . . .	160
5.6.3	High Momentum Transfers . . . . .	163
5.6.4	Relativistic Density Effect . . . . .	164
5.7	Fermi Gas . . . . .	170
5.7.1	Electronic States . . . . .	170
5.7.2	Lindhard Function . . . . .	171
5.7.3	Degenerate Fermi Gas . . . . .	172
5.7.4	Stopping Force at High Projectile Speed . . . . .	174
5.8	Discussion and Outlook . . . . .	176
	Problems . . . . .	178
	References . . . . .	179
<b>6</b>	<b>Stopping of Swift Point Charge II: Extensions</b> . . . . .	<b>181</b>
6.1	Introductory Comments . . . . .	181
6.2	Bare and Dressed Projectiles . . . . .	183
6.2.1	Bohr Screening Criterion . . . . .	183
6.3	Bloch Theory . . . . .	184
6.3.1	Bloch Formula . . . . .	184
6.3.2	Derivation . . . . .	186
6.3.3	Inverse-Bloch Correction . . . . .	188
6.3.4	Impact-Parameter Dependence . . . . .	188
6.4	Barkas-Andersen Effect . . . . .	190
6.4.1	Overview . . . . .	190
6.4.2	Dimensional Arguments . . . . .	192
6.4.3	Binding and Screening . . . . .	193
6.4.4	Higher-Order Perturbation Theory . . . . .	194
6.4.5	Beyond Perturbation Theory . . . . .	201
6.5	Stopping Medium in Internal Motion . . . . .	206
6.5.1	Nonrelativistic Regime . . . . .	206
6.5.2	Relativistic Extension (★) . . . . .	207
6.5.3	A Useful Transformation . . . . .	209
6.5.4	High-Speed Expansion: Nonrelativistic . . . . .	209
6.5.5	Relativistic Orbital Speed (★) . . . . .	210
6.6	Shell Correction . . . . .	210
6.6.1	Introduction . . . . .	210

6.6.2	Bohr Theory	211
6.6.3	Bethe Theory	212
6.6.4	Kinetic Theory	215
6.6.5	Is the Shell Correction Purely Kinematic?	216
6.7	Relativistic Projectile Speed	217
6.7.1	General Observations	217
6.7.2	Lindhard-Sørensen Theory (★)	218
6.7.3	Additional Effects	221
6.8	Discussion and Outlook	221
	Problems	222
	References	223
<b>7</b>	<b>Arriving at Numbers</b>	<b>229</b>
7.1	Introductory Comments	229
7.2	Stopping Models I: Statistical Method	230
7.2.1	Thomas-Fermi Model of the Atom	230
7.2.2	Scaling Properties	230
7.2.3	Charge and Velocity Distributions	232
7.2.4	The Lindhard-Scharff Model and its Implementation	234
7.2.5	Generalizations	239
7.3	Stopping Models II	240
7.3.1	Shell and Subshell Splitting	240
7.3.2	Kinetic Theory	243
7.3.3	Harmonic-Oscillator Model	243
7.3.4	Binary-Collision Models	246
7.3.5	Numerical Simulations	248
7.4	Remarks on Stopping Measurements	249
7.4.1	Energy-Loss Spectra in Transmission	249
7.4.2	Other Measurements on Thin Foils	250
7.4.3	Reflection Geometry	250
7.4.4	Doppler-Shift Attenuation	251
7.4.5	Pitfalls	251
7.4.6	Range Measurements	251
7.5	Extraction of Input Parameters from Stopping Measurements	251
7.5.1	$I$ -Values and Shell Correction	252
7.5.2	Barkas-Andersen and Bloch Correction	252
7.5.3	$Z_2$ Structure	254
7.6	Input Parameters from Other Sources	254
7.6.1	Theory	254
7.6.2	Optical and X-Ray Data	256
7.7	Compound Materials and Bragg Additivity	256
7.8	Data Compilations and Codes	260
7.9	Discussion and Outlook	261
	Problems	267
	References	268

---

**Part III Stragglings**


---

<b>8</b>	<b>Energy-Loss Stragglings: Variance and Higher Cumulants</b>	277
8.1	Introductory Comments	277
8.2	Classical versus Quantum Theory	278
8.3	Bohr Theory	280
8.4	Born Approximation	283
8.4.1	Harmonic oscillator	283
8.4.2	Bethe Approximation	283
8.4.3	Relativistic Extension (★)	286
8.4.4	Density Effect	288
8.5	Fermi Gas (★)	288
8.5.1	Expression for Stragglings	288
8.5.2	Static Electron Gas	289
8.5.3	Degenerate Fermi Gas: High Projectile Speed	290
8.6	Shell Correction: Kinetic Theory (★)	292
8.6.1	High-Speed Expansion	293
8.6.2	Relativistic Extension	293
8.6.3	Bohr Theory	293
8.6.4	Bethe Theory	294
8.6.5	Quantum Oscillator	294
8.6.6	Bloch Theory	294
8.6.7	Fermi Gas	295
8.6.8	Full Integration	295
8.7	Barkas-Andersen Correction (★)	295
8.8	Relativity: Lindhard-Sørensen Theory (★)	299
8.9	Bunching	300
8.9.1	Classical Estimate	300
8.9.2	Bunching in Helium	302
8.9.3	Molecular Gas	304
8.9.4	Dense Matter	307
8.10	Stragglings Measurements	312
8.10.1	Gas Targets	312
8.10.2	Solid Targets	315
8.11	Third- and Higher-Order Moments (★)	317
8.11.1	Moments and Cumulants	317
8.11.2	Free-Coulomb Scattering	318
8.11.3	Bohr Theory	318
8.11.4	Born Approximation	319
8.11.5	Relativistic Extension	320
8.11.6	Fermi Gas	321
8.11.7	Kinetic Theory	321
8.12	Discussion and Outlook	322
	Problems	322

References .....	324
<b>9 Energy-Loss Spectra .....</b>	<b>327</b>
9.1 Introductory Comments .....	327
9.2 General Aspects .....	327
9.2.1 Bothe-Landau Formula .....	329
9.2.2 Bunching .....	332
9.2.3 Moments and Cumulants to Arbitrary Order .....	332
9.2.4 Diffusion Approximation .....	333
9.2.5 An Integrable Energy-Loss Spectrum .....	334
9.3 Thin Targets .....	337
9.3.1 Bohr-Williams Approach .....	337
9.3.2 Landau's Solution .....	340
9.3.3 Lindhard's Solution (★) .....	342
9.3.4 Glazov's Solution .....	344
9.4 Moderately Thick Targets .....	346
9.4.1 Vavilov Scheme (★) .....	346
9.4.2 Method of Steepest Descent .....	349
9.4.3 Applications .....	353
9.4.4 Straight Convolution .....	355
9.5 Transport Equations .....	357
9.5.1 Derivation by Two-Layer Argument .....	357
9.5.2 Forward and Backward Equations .....	358
9.6 Very Thick Targets (★) .....	359
9.6.1 Continuous Slowing-Down Approximation .....	360
9.6.2 Ionization Yield .....	360
9.6.3 Stopping Measurement on a Thick Target .....	361
9.6.4 Straggling According to Symon (★) .....	362
9.6.5 Nonstochastic Broadening and Skewing .....	365
9.6.6 Method of Moments .....	367
9.7 Simulation .....	367
9.7.1 Monte Carlo Schemes .....	367
9.7.2 Procedure .....	368
9.7.3 Equivalence with Transport Theory (★) .....	369
9.8 Discussion and Outlook .....	369
Problems .....	371
References .....	373

---

**Part IV Appendices**


---

<b>A Selected Tutorials</b> .....	377
A.1 Units .....	377
A.1.1 Electromagnetic Units .....	377
A.1.2 Atomic Units .....	379
A.1.3 Length Measures .....	379
A.2 Calculus .....	380
A.2.1 Poisson Statistics .....	380
A.2.2 Fourier Transform .....	382
A.2.3 Spherical Harmonics and Legendre Polynomials .....	384
A.2.4 Dirac Function .....	387
A.2.5 Green Functions .....	390
A.3 Mechanics .....	395
A.3.1 Classical Perturbation Theory .....	395
A.3.2 Relativity .....	399
A.4 Quantum Mechanics .....	402
A.4.1 Gaussian Wave Packets .....	402
A.4.2 Time-Dependent Perturbation Theory .....	404
A.4.3 Generalized Oscillator Strengths for the Harmonic Oscillator .....	406
A.4.4 Sum Rules .....	408
A.4.5 Dirac Equation .....	410
A.5 Dispersion and Absorption .....	412
A.5.1 Drude Theory for a Dilute Gas .....	412
A.5.2 Quantum Theory for a Dilute Gas .....	413
A.5.3 Dense Media .....	415
A.5.4 Lindhard Function of the Fermi Gas .....	415
References .....	416
<b>B Books and Reviews</b> .....	419
References .....	422
<b>Author Index</b> .....	425
<b>Subject Index</b> .....	429