
Contents

Positivity in Natural Sciences

<i>Jacek Banasiak</i>	1
1 Introduction	1
1.1 What can go Wrong?	3
1.2 And if Everything Seems to be Fine?	3
2 Spectral Properties of Operators	4
2.1 Operators	5
2.2 Spectral Properties of a Single Operator	7
3 Banach Lattices and Positive Operators	13
3.1 Defining Order	13
3.2 Banach Lattices	15
3.3 Positive Operators	19
3.4 Relation Between Order and Norm	20
3.5 Complexification	23
3.6 Spectral Radius of Positive Operators	24
4 First Semigroups	25
4.1 Around the Hille–Yosida Theorem	27
4.2 Dissipative Operators	28
4.3 Long Time Behaviour of Semigroups	29
4.4 Positive Semigroups	37
4.5 Generation Through Perturbation	39
4.6 Positive Perturbations of Positive Semigroups	42
5 What can go Wrong?	45
5.1 Applications to Birth-and-Death Type Problems	52
5.2 Chaos in Population Theory	59
6 Asynchronous Growth	61
6.1 Essential Growth Bound	61
6.2 Peripheral Spectrum of Positive Semigroups	63
6.3 Compactness, Positivity and Irreducibility of Perturbed Semigroups	67

X Contents

7	Asymptotic Analysis of Singularly Perturbed Dynamical Systems	75
7.1	Compressed Expansion	77
References		87

Rescaling Stochastic Processes: Asymptotics

<i>V. Capasso and D. Morale</i>	91	
1	Introduction	91
1.1	First Examples of Rescaling	95
2	Stochastic Processes	97
2.1	Processes with Independent Increments	100
2.2	Martingales	100
2.3	Markov Processes	103
2.4	Brownian Motion and the Wiener Process	109
3	Itô Calculus	110
3.1	The Itô Integral	110
3.2	The Stochastic Differential	112
3.3	Stochastic Differential Equations	113
3.4	Kolmogorov and Fokker-Planck Equations	115
3.5	The Multidimensional Case	117
4	Deterministic Approximation of Stochastic Systems	118
4.1	Continuous Approximation of Jump Population Processes	118
4.2	Continuous Approximation of Stochastic Interacting Particle Systems	120
4.3	Convergence of the Empirical Measure	122
5	A Specific Model for Interacting Particles	128
5.1	Asymptotic Behavior of the System for Large Populations: A Heuristic Derivation	130
5.2	Asymptotic Behavior of the System for Large Populations: A Rigorous Derivation	134
6	Long Time Behavior: Invariant Measure	137
A	Proof of the Identification of the Limit ρ	141
References		144

**Modelling Aspects of Cancer Growth: Insight
from Mathematical and Numerical Analysis
and Computational Simulation**

<i>Mark A.J. Chaplain</i>	147	
1	Introduction	147
1.1	Macroscopic Modelling	148
1.2	Cancer Growth and Development	149
2	Modelling Avascular Solid Tumour Growth	150
2.1	Introduction	150
2.2	Linearised Stability Theory	151
2.3	The Role of Pre-Pattern Theory in Solid Tumour Growth and Invasion	153

2.4	Model Extension: Application to a Growing Spherical Tumour	156
2.5	Discussion and Conclusions	157
3	Mathematical Modelling of T-Lymphocyte Response to a Solid Tumour	160
3.1	Introduction	160
3.2	The Mathematical Model	161
3.3	Travelling Wave Analysis	173
3.4	Discussion	178
4	Mathematical Modelling of Cancer Invasion	180
4.1	Introduction	180
4.2	Cancer Invasion of Tissue and Metastasis	182
4.3	Proteolysis and Extracellular Matrix Degradation	182
4.4	The Mathematical Model of Proteolysis and Cancer Cell Invasion of Tissue	184
4.5	Nondimensionalisation of the Model Equations	187
4.6	Model Analysis	188
4.7	Spatially Uniform Steady States	188
4.8	Taxis-Driven Instability and Dispersion Curves	188
4.9	Numerical Results	189
4.10	Numerical Technique	190
4.11	Computational Simulation Results	191
4.12	Discussion and Conclusions	191
5	Summary	195
	References	195
Links Between Microscopic and Macroscopic Descriptions		
<i>Miroslaw Lachowicz</i>		201
1	Introduction	201
2	Microscopic (Stochastic) Systems	205
3	Generalized Kinetic Models	213
4	Diffusive Limit	227
5	Links in the Space–Homogeneous Case	231
6	Coagulation–Fragmentation Equations	243
7	The Space–Inhomogeneous Case: Reaction–Diffusion Equations	245
8	Reaction–Diffusion–Chemotaxis Equations	252
	References	262
Evolutionary Game Theory and Population Dynamics		
<i>Jacek Miękisz</i>		269
1	Short Overview	269
2	Introduction	270
3	A Crash Course in Game Theory	273
4	Replicator Dynamics	277
5	Replicator Dynamics with Migration	280

XII Contents

6	Replicator Dynamics with Time Delay	285
6.1	Social-Type Time Delay	285
6.2	Biological-Type Time Delay	288
7	Stochastic Dynamics of Finite Populations.....	290
8	Stochastic Dynamics of Well-Mixed Populations	292
9	Spatial Games with Local Interactions	298
9.1	Nash Configurations and Stochastic Dynamics	298
9.2	Ground States and Nash Configurations	300
9.3	Ensemble Stability	303
9.4	Stochastic Stability in Non-Potential Games.....	306
9.5	Dominated Strategies	310
10	Review of Other Results	311
	References	312
	List of Participants	317
	Index	319