
Contents

1	Biomedical Imaging Modalities	3
1.1	X-Ray Imaging and Computed Tomography	3
1.2	Magnetic Resonance Imaging	4
1.3	Electrical Impedance Tomography	5
1.4	T-Scan Electrical Impedance Imaging System for Anomaly Detection.....	7
1.5	Electrical and Magnetic Source Imaging	7
1.6	Magnetic Resonance Electrical Impedance Tomography	9
1.7	Impediography	10
1.8	Ultrasound Imaging.....	11
1.9	Microwave Imaging	12
1.10	Elastic Imaging	12
1.11	Magnetic Resonance Elastography	12
1.12	Optical Tomography	13

Part I Mathematical Tools

2	Preliminaries	17
2.1	Special Functions.....	17
2.2	Sobolev Spaces	20
2.3	Fourier Analysis.....	21
2.3.1	Shannon's Sampling Theorem.....	23
2.3.2	Fast Fourier Transform.....	24
2.4	The Two-Dimensional Radon Transform.....	25
2.5	The Moore-Penrose Generalized Inverse	28
2.6	Singular Value Decomposition.....	28
2.7	Compact Operators.....	29
2.8	Regularization of Ill-Posed Problems	30
2.8.1	Stability	30
2.8.2	The Truncated SVD	32

2.8.3	Tikhonov-Phillips Regularization	32
2.8.4	Regularization by Truncated Iterative Methods	34
2.9	General Image Characteristics	35
2.9.1	Spatial Resolution	35
2.9.2	Signal-To-Noise Ratio	37
3	Layer Potential Techniques	43
3.1	The Laplace Equation	44
3.1.1	Fundamental Solution	44
3.1.2	Layer Potentials	46
3.1.3	Invertibility of $\lambda I - \mathcal{K}_D^*$	54
3.1.4	Neumann Function	55
3.1.5	Transmission Problem	59
3.2	Helmholtz Equation	62
3.2.1	Fundamental Solution	62
3.2.2	Layer Potentials	63
3.2.3	Transmission Problem	65
3.3	Static Elasticity	70
3.3.1	Fundamental Solution	71
3.3.2	Layer Potentials	73
3.3.3	Transmission Problem	75
3.4	Dynamic Elasticity	80
3.4.1	Radiation Condition	81
3.4.2	Fundamental Solution	81
3.4.3	Layer Potentials	82
3.4.4	Transmission Problem	83
3.5	Modified Stokes System	84
3.5.1	Fundamental Solution	84
3.5.2	Layer Potentials	85
3.5.3	Transmission Problem	89

Part II General Reconstruction Algorithms

4	Tomographic Imaging with Non-Diffracting Sources	95
4.1	Imaging Equations of CT and MRI	95
4.1.1	Imaging Equation of CT	95
4.1.2	Imaging Equation of MRI	96
4.2	General Issues of Image Reconstruction	97
4.3	Reconstruction from Fourier Transform Samples	98
4.3.1	Problem Formulation	98
4.3.2	Basic Theory	99
4.4	Reconstruction from Radon Transform Samples	101
4.4.1	The Inverse Radon Transform	101
4.4.2	Fourier Inversion Formula	101

4.4.3	Direct Backprojection Method	102
4.4.4	Filtered Backprojection Reconstruction	104
4.4.5	Noise in Filtered Backprojection Reconstruction	105
5	Tomographic Imaging with Diffracting Sources	107
5.1	Electrical Impedance Tomography	107
5.1.1	Mathematical Model	108
5.1.2	Ill-Conditioning	108
5.1.3	Static Imaging	109
5.1.4	Dynamic Imaging	110
5.1.5	Electrode Model	112
5.2	Ultrasound and Microwave Tomographies	112
5.2.1	Mathematical Model	113
5.2.2	Diffraction Tomography	114
6	Biomagnetic Source Imaging	117
6.1	Mathematical Models	118
6.1.1	The Electric Forward Problem	119
6.1.2	The Magnetic Forward Problem	119
6.2	The Inverse EEG Problem	120
6.3	The Spherical Model in MEG	121

Part III Anomaly Detection Algorithms

7	Small Volume Expansions	127
7.1	Conductivity Problem	128
7.1.1	Formal Derivations	129
7.1.2	Polarization Tensor	131
7.2	Helmholtz Equation	132
7.2.1	Formal Derivations	134
7.3	Static Elasticity	134
7.3.1	Formal Derivations	136
7.3.2	Elastic Moment Tensor	138
7.4	Dynamic Elasticity	140
7.5	Modified Stokes System	140
7.6	Nearly Incompressible Bodies	141
7.6.1	Formal Derivations	142
7.6.2	Viscous Moment Tensor	145
7.7	Diffusion Equation	147
8	Imaging Techniques	151
8.1	Projection Type Algorithms	151
8.2	Multiple Signal Classification Type Algorithms	152
8.3	Time-Domain Imaging	156

8.3.1 Fourier- and MUSIC-Type Algorithms 157
8.3.2 Time-Reversal Imaging 159

Part IV Hybrid Imaging Techniques

9 Magnetic Resonance Electrical Impedance Tomography 169
9.1 Mathematical Model 170
9.2 J-Substitution Algorithm 172
9.3 The Harmonic Algorithm 174

10 Impediography 177
10.1 Physical Model 177
10.2 Mathematical Model 178
10.3 E-Substitution Algorithm 180

11 Magnetic Resonance Elastography 183
11.1 Mathematical Model 183
11.2 Binary Level Set Algorithm 185

References 189

Index 197