

Table of Contents

Part I. Nonrelativistic Many-Particle Systems

1. Second Quantization	3
1.1 Identical Particles, Many-Particle States, and Permutation Symmetry	3
1.1.1 States and Observables of Identical Particles	3
1.1.2 Examples	6
1.2 Completely Symmetric and Antisymmetric States	8
1.3 Bosons	10
1.3.1 States, Fock Space, Creation and Annihilation Operators	10
1.3.2 The Particle-Number Operator	13
1.3.3 General Single- and Many-Particle Operators	14
1.4 Fermions	16
1.4.1 States, Fock Space, Creation and Annihilation Operators	16
1.4.2 Single- and Many-Particle Operators	19
1.5 Field Operators	20
1.5.1 Transformations Between Different Basis Systems	20
1.5.2 Field Operators	21
1.5.3 Field Equations	23
1.6 Momentum Representation	25
1.6.1 Momentum Eigenfunctions and the Hamiltonian	25
1.6.2 Fourier Transformation of the Density	27
1.6.3 The Inclusion of Spin	27
Problems	29
2. Spin-1/2 Fermions	33
2.1 Noninteracting Fermions	33
2.1.1 The Fermi Sphere, Excitations	33
2.1.2 Single-Particle Correlation Function	35
2.1.3 Pair Distribution Function	36
*2.1.4 Pair Distribution Function, Density Correlation Functions, and Structure Factor	39

2.2	Ground State Energy and Elementary Theory of the Electron Gas	41
2.2.1	Hamiltonian	41
2.2.2	Ground State Energy in the Hartree–Fock Approximation.....	42
2.2.3	Modification of Electron Energy Levels due to the Coulomb Interaction	46
2.3	Hartree–Fock Equations for Atoms	49
	Problems	52
3.	Bosons	55
3.1	Free Bosons.....	55
3.1.1	Pair Distribution Function for Free Bosons	55
*3.1.2	Two-Particle States of Bosons.....	57
3.2	Weakly Interacting, Dilute Bose Gas	60
3.2.1	Quantum Fluids and Bose–Einstein Condensation	60
3.2.2	Bogoliubov Theory of the Weakly Interacting Bose Gas.....	62
*3.2.3	Superfluidity.....	69
	Problems	72
4.	Correlation Functions, Scattering, and Response	75
4.1	Scattering and Response	75
4.2	Density Matrix, Correlation Functions	82
4.3	Dynamical Susceptibility	85
4.4	Dispersion Relations	89
4.5	Spectral Representation	90
4.6	Fluctuation–Dissipation Theorem.....	91
4.7	Examples of Applications.....	93
*4.8	Symmetry Properties	100
4.8.1	General Symmetry Relations.....	100
4.8.2	Symmetry Properties of the Response Function for Hermitian Operators.....	102
4.9	Sum Rules	107
4.9.1	General Structure of Sum Rules	107
4.9.2	Application to the Excitations in He II	108
	Problems	109
	Bibliography for Part I	111

Part II. Relativistic Wave Equations

5. Relativistic Wave Equations and their Derivation	115
5.1 Introduction	115
5.2 The Klein–Gordon Equation	116
5.2.1 Derivation by Means of the Correspondence Principle .	116
5.2.2 The Continuity Equation	119
5.2.3 Free Solutions of the Klein–Gordon Equation	120
5.3 Dirac Equation	120
5.3.1 Derivation of the Dirac Equation	120
5.3.2 The Continuity Equation	122
5.3.3 Properties of the Dirac Matrices	123
5.3.4 The Dirac Equation in Covariant Form	123
5.3.5 Nonrelativistic Limit and Coupling to the Electromagnetic Field	125
Problems	130
6. Lorentz Transformations and Covariance of the Dirac Equation	131
6.1 Lorentz Transformations	131
6.2 Lorentz Covariance of the Dirac Equation	135
6.2.1 Lorentz Covariance and Transformation of Spinors....	135
6.2.2 Determination of the Representation $S(\mathbf{A})$	136
6.2.3 Further Properties of S	142
6.2.4 Transformation of Bilinear Forms	144
6.2.5 Properties of the γ Matrices	145
6.3 Solutions of the Dirac Equation for Free Particles	146
6.3.1 Spinors with Finite Momentum	146
6.3.2 Orthogonality Relations and Density	149
6.3.3 Projection Operators	151
Problems	152
7. Orbital Angular Momentum and Spin	155
7.1 Passive and Active Transformations	155
7.2 Rotations and Angular Momentum	156
Problems	159
8. The Coulomb Potential	161
8.1 Klein–Gordon Equation with Electromagnetic Field	161
8.1.1 Coupling to the Electromagnetic Field	161
8.1.2 Klein–Gordon Equation in a Coulomb Field	162
8.2 Dirac Equation for the Coulomb Potential	168
Problems	180

9. The Foldy–Wouthuysen Transformation and Relativistic Corrections	181
9.1 The Foldy–Wouthuysen Transformation	181
9.1.1 Description of the Problem	181
9.1.2 Transformation for Free Particles	182
9.1.3 Interaction with the Electromagnetic Field	183
9.2 Relativistic Corrections and the Lamb Shift	187
9.2.1 Relativistic Corrections	187
9.2.2 Estimate of the Lamb Shift	189
Problems	193
10. Physical Interpretation of the Solutions to the Dirac Equation	195
10.1 Wave Packets and “Zitterbewegung”	195
10.1.1 Superposition of Positive Energy States	196
10.1.2 The General Wave Packet	197
*10.1.3 General Solution of the Free Dirac Equation in the Heisenberg Representation	200
*10.1.4 Potential Steps and the Klein Paradox	202
10.2 The Hole Theory	204
Problems	207
11. Symmetries and Further Properties of the Dirac Equation	209
*11.1 Active and Passive Transformations, Transformations of Vectors	209
11.2 Invariance and Conservation Laws	212
11.2.1 The General Transformation	212
11.2.2 Rotations	212
11.2.3 Translations	213
11.2.4 Spatial Reflection (Parity Transformation)	213
11.3 Charge Conjugation	214
11.4 Time Reversal (Motion Reversal)	217
11.4.1 Reversal of Motion in Classical Physics	218
11.4.2 Time Reversal in Quantum Mechanics	221
11.4.3 Time-Reversal Invariance of the Dirac Equation	229
*11.4.4 Racah Time Reflection	235
*11.5 Helicity	236
*11.6 Zero-Mass Fermions (Neutrinos)	239
Problems	244
Bibliography for Part II	245

Part III. Relativistic Fields

12. Quantization of Relativistic Fields	249
12.1 Coupled Oscillators, the Linear Chain, Lattice Vibrations	249
12.1.1 Linear Chain of Coupled Oscillators	249
12.1.2 Continuum Limit, Vibrating String	255
12.1.3 Generalization to Three Dimensions, Relationship to the Klein–Gordon Field	258
12.2 Classical Field Theory	261
12.2.1 Lagrangian and Euler–Lagrange Equations of Motion	261
12.3 Canonical Quantization	266
12.4 Symmetries and Conservation Laws, Noether’s Theorem	266
12.4.1 The Energy–Momentum Tensor, Continuity Equations, and Conservation Laws	266
12.4.2 Derivation from Noether’s Theorem of the Conservation Laws for Four-Momentum, Angular Momentum, and Charge	268
Problems	275
13. Free Fields	277
13.1 The Real Klein–Gordon Field	277
13.1.1 The Lagrangian Density, Commutation Relations, and the Hamiltonian	277
13.1.2 Propagators	281
13.2 The Complex Klein–Gordon Field	285
13.3 Quantization of the Dirac Field	287
13.3.1 Field Equations	287
13.3.2 Conserved Quantities	289
13.3.3 Quantization	290
13.3.4 Charge	293
*13.3.5 The Infinite-Volume Limit	295
13.4 The Spin Statistics Theorem	296
13.4.1 Propagators and the Spin Statistics Theorem	296
13.4.2 Further Properties of Anticommutators and Propagators of the Dirac Field	301
Problems	303
14. Quantization of the Radiation Field	307
14.1 Classical Electrodynamics	307
14.1.1 Maxwell Equations	307
14.1.2 Gauge Transformations	309
14.2 The Coulomb Gauge	309
14.3 The Lagrangian Density for the Electromagnetic Field	311
14.4 The Free Electromagnetic Field and its Quantization	312

14.5 Calculation of the Photon Propagator	316
Problems	320
15. Interacting Fields, Quantum Electrodynamics	321
15.1 Lagrangians, Interacting Fields	321
15.1.1 Nonlinear Lagrangians	321
15.1.2 Fermions in an External Field	322
15.1.3 Interaction of Electrons with the Radiation Field: Quantum Electrodynamics (QED)	322
15.2 The Interaction Representation, Perturbation Theory	323
15.2.1 The Interaction Representation (Dirac Representation)	324
15.2.2 Perturbation Theory	327
15.3 The S Matrix	328
15.3.1 General Formulation	328
15.3.2 Simple Transitions	332
*15.4 Wick's Theorem	335
15.5 Simple Scattering Processes, Feynman Diagrams	339
15.5.1 The First-Order Term	339
15.5.2 Mott Scattering	341
15.5.3 Second-Order Processes	346
15.5.4 Feynman Rules of Quantum Electrodynamics	356
*15.6 Radiative Corrections	358
15.6.1 The Self-Energy of the Electron	359
15.6.2 Self-Energy of the Photon, Vacuum Polarization	365
15.6.3 Vertex Corrections	366
15.6.4 The Ward Identity and Charge Renormalization	368
15.6.5 Anomalous Magnetic Moment of the Electron	371
Problems	373
Bibliography for Part III	375
Appendix	377
A Alternative Derivation of the Dirac Equation	377
B Dirac Matrices	379
B.1 Standard Representation	379
B.2 Chiral Representation	379
B.3 Majorana Representations	380
C Projection Operators for the Spin	380
C.1 Definition	380
C.2 Rest Frame	380
C.3 General Significance of the Projection Operator $P(n)$	381
D The Path-Integral Representation of Quantum Mechanics	385
E Covariant Quantization of the Electromagnetic Field, the Gupta–Bleuler Method	387
E.1 Quantization and the Feynman Propagator	387

E.2	The Physical Significance of Longitudinal and Scalar Photons	389
E.3	The Feynman Photon Propagator	392
E.4	Conserved Quantities	393
F	Coupling of Charged Scalar Mesons to the Electromagnetic Field	394
Index	397