

Contents

Part I Non-Associative and Non-Commutative Structures for Physics

1	Moufang Transformations and Noether Currents	3
	Eugen Paal	
1.1	Introduction	3
1.2	Moufang Loops and Mal'tsev Algebras	4
1.3	Birepresentations	4
1.4	Moufang–Noether Currents and ETC	6
	References	8
2	Weakly Nonassociative Algebras, Riccati and KP Hierarchies	9
	Aristophanes Dimakis and Folkert Müller-Hoissen	
2.1	Introduction	9
2.2	Nonassociativity and KP	10
2.3	A Class of WNA Algebras and a Matrix Riccati Hierarchy	13
2.4	WNA Algebras and Solutions of the Discrete KP Hierarchy	17
2.5	From WNA to Gelfand–Dickey–Sato	20
2.6	Conclusions	23
	References	24
3	Applications of Transvectants	29
	Chris Athorne	
3.1	Introduction	29
3.2	Transvectants	30
3.3	Hirota	31
3.4	Padé	33
3.5	Hyperelliptic	34
	References	36

4	Automorphisms of Finite Orthoalgebras, Exceptional Root Systems and Quantum Mechanics	39
	Artur E. Ruuge and Fred Van Oystaeyen	
4.1	Introduction	39
4.2	Saturated Configurations	41
4.3	Non-Colourable Configurations	41
4.4	The E_6 Case	42
4.5	Orthoalgebras Generated by E_8	43
4.6	Conclusions	45
	References	45
5	A Rewriting Approach to Graph Invariants	47
	Lars Hellström	
5.1	Background	47
5.2	Graph Theory	48
5.3	The Problem	50
5.4	Semigraphs	52
5.5	Applying the Diamond Lemma	58
5.6	Classification of Invariants	64
	References	67
Part II Non-Commutative Deformations, Quantization, Homological Methods, and Representations		
6	Graded q-Differential Algebra Approach to q-Connection	71
	Viktor Abramov	
6.1	Introduction	71
6.2	Graded q -Differential Algebra	72
6.3	q -Connection and Its Curvature	73
6.4	Matrix of a q -Connection	75
	References	79
7	On Generalized N-Complexes Coming from Twisted Derivations	81
	Daniel Larsson and Sergei D. Silvestrov	
7.1	Introduction	81
7.2	General Framework of (σ, τ) -Derivations	82
7.3	Generalized N -Complexes and an Example	86
	References	88
8	Remarks on Quantizations, Words and R-Matrices	89
	Hilja L. Huru	
8.1	Introduction	89
8.2	Multiplicative Cohomologies of Monoids	90
8.3	Graded Modules	92
8.4	Letters and Words	94

Contents	xi
8.5 Quantizations of R -Matrices	95
References	98
9 Connections on Modules over Singularities of Finite and Tame CM Representation Type	99
Eivind Eriksen and Trond Stølen Gustavsen	
9.1 Introduction	99
9.2 Preliminaries	100
9.3 Obstruction Theory	102
9.4 Results and Examples	104
References	107
10 Computing Noncommutative Global Deformations of D-Modules	109
Eivind Eriksen	
10.1 Introduction	109
10.2 Noncommutative Global Deformations of D-Modules	110
10.3 Computing Noncommutative Global Deformations	111
10.4 Calculations for D-Modules on Elliptic Curves	113
References	117
11 Comparing Small Orthogonal Classes	119
Gabriella D'Este	
11.1 Introduction	119
11.2 Preliminaries	120
11.3 Proofs and Examples	122
References	128
Part III Groups and Actions	
12 How to Compose Lagrangian?	131
Eugen Paal and Jüri Virkepu	
12.1 Introduction	131
12.2 General Method for Constructing Lagrangians	132
12.3 Lagrangian for $SO(2)$	133
12.4 Physical Interpretation	136
12.5 Lagrangian for the Affine Transformations of the Line	136
References	140
13 Semidirect Products of Generalized Quaternion Groups by a Cyclic Group	141
Peeter Puusemp	
13.1 Introduction	141
13.2 Semidirect Products of Q_n by C_2	142
13.3 A Description of \mathcal{G}_1 , \mathcal{G}_2 and \mathcal{G}_3 by Their Endomorphisms	146
References	149

14	A Characterization of a Class of 2-Groups by Their Endomorphism Semigroups	151
	Tatjana Gramushnjak and Peeter Puusemp	
14.1	Introduction	151
14.2	The Group G_{17}	153
14.3	The Group G_{20}	155
14.4	The Group G_{27}	156
	References	158
15	Adjoint Representations and Movements	161
	Maido Rahula and Vitali Retšnoi	
15.1	Introduction	161
15.2	Generalized Leibnitz Rule	162
15.3	Tangent Group	162
15.4	Linear Group $GL(2, \mathbb{R})$	163
15.5	The Operator of Center	165
15.6	Discriminant Parabola	166
15.7	Relations to Moments in Probability Theory	167
15.8	Conclusion	169
	References	170
16	Applications of Hypocontinuous Bilinear Maps in Infinite-Dimensional Differential Calculus	171
	Helge Glöckner	
16.1	Introduction	171
16.2	Preliminaries and Basic Facts	172
16.3	Differentiability Properties of Compositions with Hypocontinuous Bilinear Mappings	178
16.4	Holomorphic Families of Operators	180
16.5	Locally Convex Poisson Vector Spaces	182
	References	186
Part IV Quasi-Lie, Super-Lie, Hom-Hopf and Super-Hopf Structures and Extensions, Deformations and Generalizations of Infinite-Dimensional Lie Algebras		
17	Hom-Lie Admissible Hom-Coalgebras and Hom-Hopf Algebras	189
	Abdenacer Makhlouf and Sergei Silvestrov	
17.1	Introduction	189
17.2	Hom-Algebra and Hom-Coalgebra Structures	191
17.3	Hom-Lie Admissible Hom-Coalgebras	193
17.4	Hom-Hopf Algebras	199
	References	205

18	Bosonisation and Parastatistics	207
	K. Kanakoglou and C. Daskaloyannis	
	18.1 Introduction and Definitions	207
	18.2 (Super-)Lie and (Super-)Hopf Algebraic Structure of the Parabosonic $P_B^{(n)}$ and Parafermionic $P_F^{(n)}$ Algebras	208
	18.3 Bosonisation as a Technique of Reducing Supersymmetry	212
	18.4 Discussion	217
	References	218
19	Deformations of the Witt, Virasoro, and Current Algebra	219
	Martin Schlichenmaier	
	19.1 Introduction	219
	19.2 Deformations of Lie Algebras	221
	19.3 Krichever–Novikov Algebras	223
	19.4 The Geometric Families	226
	19.5 The Geometric Background	229
	19.6 Examples for the Degenerated Situations	230
	References	233
20	Conformal Algebras in the Context of Linear Algebraic Groups	235
	Pavel Kolesnikov	
	20.1 Introduction	235
	20.2 Categories of Conformal Algebras	237
	20.3 Associative (G)-Conformal Algebras	240
	20.4 Conformal Endomorphism Algebra over a Linear Algebraic Group	243
	References	246
21	Lie Color and Hom-Lie Algebras of Witt Type and Their Central Extensions	247
	Gunnar Sigurdsson and Sergei Silvestrov	
	21.1 Introduction	247
	21.2 Central Extensions of Witt-Type Lie Color Algebras	248
	21.3 Central Extensions of Γ -Graded Hom-Lie Algebras of Witt Type	252
	References	254
22	A Note on Quasi-Lie and Hom-Lie Structures of σ-Derivations of $\mathbb{C}[z_1^{\pm 1}, \dots, z_n^{\pm 1}]$	257
	Lionel Richard and Sergei Silvestrov	
	22.1 Introduction	257
	22.2 Framework	258
	22.3 Sufficient Condition	260
	22.4 Laurent Polynomials	261
	References	262

Part V Commutative Subalgebras in Noncommutative Algebras

23 Algebraic Dependence of Commuting Elements in Algebras	265
Sergei Silvestrov, Christian Svensson, and Marcel de Jeu	
23.1 Introduction	265
23.2 Description of the Problem: Commuting Elements in an Algebra Are Given, Then Find Curves They Lie on	267
23.3 Burchnell–Chaundy Construction for Differential Operators	269
23.4 Burchnell–Chaundy Theory for the q -Deformed Heisenberg Algebra	273
References	279
24 Crossed Product-Like and Pre-Crystalline Graded Rings	281
Johan Öinert and Sergei D. Silvestrov	
24.1 Introduction	281
24.2 Preliminaries and Definitions	282
24.3 The Commutant of \mathcal{A}_0 in a Crossed Product-Like Ring	284
24.4 The Center of a Crossed Product-Like Ring $\mathcal{A}_0 \diamond_{\sigma}^{\alpha} M$	286
24.5 Intersection Theorems	288
24.6 Examples of Crossed Product-Like and Crystalline Graded Rings	292
References	295
25 Decomposition of the Enveloping Algebra $\mathfrak{so}(5)$	297
Čestmír Burdík and Ondřej Navrátil	
25.1 Introduction	297
25.2 The Lie Algebra $\mathfrak{so}(5)$	298
25.3 The Highest Weight Vectors	299
25.4 Conclusion	302
References	302
Index	303