

Contents

Introduction	1
I Locally Convex Cones	9
1. Locally Convex Cones	10
Locally Convex Cones via Convex Quasiuniform Structures ..	13
2. Continuous Linear Operators and Functionals	17
Embeddings	20
3. Weak Local and Global Preorders	23
4. Boundedness and the Relative Topologies	26
The Weak Topology $\sigma(\mathcal{P}, \mathcal{P}^*)$	31
Boundedness Components	32
Connectedness	41
Locally Convex Cones with Uniform Boundedness Components	42
Bounded Subsets	42
Closed Convex Sets	44
5. Locally Convex Lattice Cones	56
Locally Convex Lattice Cones	57
Locally Convex Complete Lattice Cones	58
Zero Components	62
Order Convergence	69
Series	74
Order Continuous Linear Operators and Functionals	77
Lattice Homomorphisms	77
Functionals Supporting the Separation Property	79
Almost Order Convergent Nets	88
Order Topology	91
Weak Order Convergence	93
Extension of Linear Operators	96
The Standard Lattice Completion of a Locally Convex Cone .	97
Simplified Standard Lattice Completion	104
6. Quasi-Full Locally Convex Cones	107
Quasi-Full Locally Convex Cones	107
The Standard Full Extension of a Quasi-Full Cone	109

7.	Cones of Linear Operators	111
	Cones of Linear Functionals. The Second Dual	114
8.	Notes and Remarks	116
II	Measures and Integrals. The General Theory	119
1.	Measurable Cone-Valued Functions	119
	Weak σ -Rings	120
	Measurable Functions	120
2.	Inductive Limit Neighborhoods for Cone-Valued Functions	127
	Infinity as a Neighborhood	127
	Inductive Limit Neighborhoods	127
	The Cone $\mathcal{F}_{\mathfrak{A}}(X, \mathcal{P})$	128
3.	Operator-Valued Measures	131
	The Modulus of a Measure	132
	Bounded Measures	134
	Extension of a Measure	136
	Composition of Measures and Continuous Linear Operators	137
	Strong Additivity	138
	Weak Compactness	141
4.	Integrals for Cone-Valued Functions	141
	Integrals for \mathcal{P} -Valued Step Functions	143
	Integrals for Functions in $\mathcal{F}_{\mathfrak{A}}(X, \mathcal{P})$	147
	Sets of Measure Zero and Properties Holding	
	Almost Everywhere	149
	Integrability over a Set $E \in \mathfrak{A}$	150
	Integrability over a Set $F \in \mathfrak{A}_{\mathfrak{A}}$	150
5.	The General Convergence Theorems	159
	Families of Measures and Properties Holding	
	Almost Everywhere	159
	Equibounded Families of Measures	159
	Integrability with Respect to Equibounded Families	
	of Measures	159
	The Locally Convex Cone $(\mathcal{F}_{(F, \Theta)}(X, \mathcal{P}), \mathfrak{A}(F, \Theta))$	160
	Subcone-Based Integrability	161
	Sums, Multiples and Order for Measures	171
	Convergence of Sequences of Measures	174
	Residual Components	176
	Strongly Integrable Functions	178
	Convergence of Sequences in $\mathcal{F}(X, \mathcal{P})$	182
	Remarks	192
	Strong Additivity	194
	Weakly Sequentially Compact Sets of Measures	197
6.	Examples and Special Cases	207
	The case $\mathcal{Q} = \mathbb{R}$	208
	Extended Positive-Valued Functions and Measures	208

Extended Real-Valued Functions and Positive-Valued Measures	209
Real- or Complex-Valued Functions and Measures	209
The Case that \mathcal{Q} Is the Standard Lattice Completion of Some Subcone \mathcal{Q}_0	211
Compact and Weakly Compact Measures	214
The Case that \mathcal{P} Is a Locally Convex Vector Space	216
Algebra Homomorphisms	220
Lattice Homomorphisms	224
Cone-Valued Functions and Positive Real-Valued Measures . .	228
Vector-Valued Functions and Real- or Complex-Valued Measures	229
Operator-Valued Functions and Operator-Valued Measures . .	231
Positive, Real or Complex-Valued Functions and Operator-Valued Measures	234
Operator-Valued Functions and Cone-Valued Measures	238
Positive, Real or Complex-Valued Functions and Cone- or Vector-Valued Measures	239
Positive Linear Operators on Cones of $\overline{\mathbb{R}}$ -Valued Functions . .	242
Bounded Linear Operators on Spaces of Real- or Complex-Valued Functions	244
7. Extended Integrability	245
8. Notes and Remarks	246
III Measures on Locally Compact Spaces	249
1. Relatively Continuous Cone-Valued Functions	249
Elementary Functions	256
2. Cone-Valued Functions on Locally Compact Spaces	257
Inductive Limit Topologies	257
Functions that Vanish at Infinity	258
The Cones $\mathcal{E}(X, \mathcal{P})$ and $\mathcal{E}_0(X, \mathcal{P})$	258
The Cones $\mathcal{F}_{\mathfrak{A}}(X, \mathcal{P})$ and $\mathcal{F}_{\mathfrak{A}_0}(X, \mathcal{P})$	258
The Cones $\mathcal{C}_{\mathfrak{A}}^r(X, \mathcal{P})$ and $\mathcal{C}_{\mathfrak{A}_0}^r(X, \mathcal{P})$	259
3. Continuous Linear Operators on Cones of Functions	266
4. Measures on Locally Compact Spaces	273
Regularity of Measures	274
Measures as Continuous Linear Operators	282
5. Integral Representation	286
6. Special Cases and Applications	307
The Case that \mathcal{Q} Is the Standard Lattice Completion of Some Subcone \mathcal{Q}_0	307
Compact and Weakly Compact Operators	307
Locally Convex Topological Vector Spaces	314
Algebra Homomorphisms	318
Lattice Homomorphisms	325

The Case $\mathcal{P} = \overline{\mathbb{R}}$	330
The Case $\mathcal{P} = \overline{\mathbb{R}}_+$	330
The Case $\mathcal{Q} = \overline{\mathbb{R}}$	330
Sequence Cones	331
The Convergence Theorems	331
The Case that \mathcal{Q} Is the Standard Lattice Completion of Some Operator Cone	332
The Spectral Theorem	338
7. Notes and Remarks	338
List of Symbols	341
References	345
Index	353