

Contents

Preface	ix
 Part I Basic Techniques	
1 Hilbert Space Techniques	
1.1 The projection on a closed convex set	3
1.2 The Riesz representation theorem	6
1.3 The Lax–Milgram theorem	8
1.4 Convergence techniques	10
Exercises	11
2 A Survey of Essential Analysis	
2.1 L^p -techniques	13
2.2 Introduction to distributions	18
2.3 Sobolev Spaces	22
Exercises	32
3 Weak Formulation of Elliptic Problems	
3.1 Motivation	35
3.2 The weak formulation	38
Exercises	41
4 Elliptic Problems in Divergence Form	
4.1 Weak formulation	43
4.2 The weak maximum principle	49
4.3 Inhomogeneous problems	53
Exercises	54
5 Singular Perturbation Problems	
5.1 A prototype of a singular perturbation problem	57
5.2 Anisotropic singular perturbation problems	61
Exercises	69

6 Problems in Large Cylinders	
6.1 A model problem	73
6.2 Another type of convergence	79
6.3 The general case	82
6.4 An application	86
Exercises	89
7 Periodic Problems	
7.1 A general theory	93
7.2 Some additional remarks	101
Exercises	103
8 Homogenization	
8.1 More on periodic functions	106
8.2 Homogenization of elliptic equations	109
8.2.1 The one-dimensional case	109
8.2.2 The n -dimensional case	112
Exercises	119
9 Eigenvalues	
9.1 The one-dimensional case	121
9.2 The higher-dimensional case	123
9.3 An application	127
Exercises	128
10 Numerical Computations	
10.1 The finite difference method	129
10.2 The finite element method	135
Exercises	147
Part II More Advanced Theory	
11 Nonlinear Problems	
11.1 Monotone methods	153
11.2 Quasilinear equations	160
11.3 Nonlocal problems	166
11.4 Variational inequalities	170
Exercises	174

12 L^∞-estimates	
12.1 Some simple cases	177
12.2 A more involved estimate	180
12.3 The Sobolev–Gagliardo–Nirenberg inequality	183
12.4 The maximum principle on small domains	187
Exercises	188
13 Linear Elliptic Systems	
13.1 The general framework	191
13.2 Some examples	197
Exercises	202
14 The Stationary Navier–Stokes System	
14.1 Introduction	203
14.2 Existence and uniqueness result	205
Exercises	208
15 Some More Spaces	
15.1 Motivation	211
15.2 Essential features of the Sobolev spaces $W^{k,p}$	212
15.3 An application	215
Exercises	216
16 Regularity Theory	
16.1 Introduction	217
16.2 The translation method	221
16.3 Regularity of functions in Sobolev spaces	224
16.4 The bootstrap technique	227
Exercises	229
17 The p-Laplace Equation	
17.1 A minimization technique	231
17.2 A weak maximum principle and its consequences	237
17.3 A generalization of the Lax–Milgram theorem	239
Exercises	245
18 The Strong Maximum Principle	
18.1 A first version of the maximum principle	247
18.2 The Hopf maximum principle	252
18.3 Application: the moving plane technique	255
Exercises	259

19 Problems in the Whole Space

19.1	The harmonic functions, Liouville theorem	261
19.2	The Schrödinger equation	267
	Exercises	273

Appendix: Fixed Point Theorems

A.1	The Brouwer fixed point theorem	275
A.2	The Schauder fixed point theorem	279
	Exercises	280

Bibliography	281
---------------------	-----------	-----

Index	287
--------------	-----------	-----